Mobile Crowd Sensing via Online Communities: Incentive Mechanisms for Multiple Cooperative Tasks

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Abstract—Mobile crowd sensing emerges as a new paradigm which takes advantage of the pervasive sensor-embedded smartphones to collect data efficiently. Many incentive mechanisms for mobile crowd sensing have been proposed. However, none of them has taken into consideration the cooperative compatibility of users for multiple cooperative tasks. In this paper, we design truthful incentive mechanisms to minimize the social cost such that each of the cooperative tasks can be completed by a group of compatible users. We consider that the mobile crowd sensing is launched in an online community. We study two bid models and formulated the Social Optimization Compatible User Selection (SOCUS) problem for each model. We also define three compatibility models and explore the compatible relation via the social relation of online communities. We design two reverse auction based incentive mechanisms, MCT-M and MCT-S. Both of them consist of two steps: compatible user grouping and reverse auction. Through both rigid theoretical analysis and extensive simulations, we demonstrate that the proposed mechanisms achieve computational efficiency, individual rationality and truthfulness. In addition, MCT-M can output the optimal solution.

Keywords—Mobile crowd sensing; Incentive mechanism design; Online community; Compatibility

I. INTRODUCTION

Smartphones are widely available in the recent years. The worldwide smartphone market reached a total of 1.45 billion units shipped in 2016. From there, shipments will reach 1.71 billion units in 2020 [1]. Nowadays, smartphones are integrated with a variety of sensors such as camera, light sensor, GPS, accelerometer, digital compass, gyroscope, microphone, and proximity sensor. These sensors can collectively monitor a diverse range of human activities and surrounding environment. Mobile crowd sensing has become an efficient approach to meeting the demands in large scale sensing applications [2], such as Sensorly [3] for 3G/WiFi discovery, TrMCD [4] for estimating user motion trajectory, crowd-participated system [5] for bus arrival time prediction, participAct [6] for urban crowdsensing, etc.

Incentive mechanisms are crucial to mobile crowd sensing while the smartphone users spend their time and consume battery, memory, computing power and data traffic of device to sense, store and transmit the data. Moreover, there are potential privacy threats to smartphone users by sharing their sensed data with location tags, interests or identities. Incentive mechanisms are also helpful to improve sensing service quality. A lot of research efforts have been focused on designing incentive mechanisms to entice users to participate in mobile crowd sensing system. However, they either focus on the multiple independent task scenario [7, 9-17], where each task only needs one user to perform, or pay attention to the single cooperative task scenario [8, 18], where the task requires a group of users to perform cooperatively. [19, 24] has designed an incentive mechanism for multiple cooperative tasks, however, they don’t consider the relation among users.

The multiple cooperative task scenarios are very common in mobile crowd sensing. For example, the construct of fingerprint database [4] requires enough users to report sensor readings, such that the correctness of trajectory can be guaranteed by the statistical method. In the bus arrival time prediction system [5], insufficient amount of uploaded information may result in inaccuracy in matching the bus route. Many time window dependent crowd sensing tasks [8], such as continuous measure of trace, traffic condition, noise and air pollution need a large sample space such that its result has the statistical meanings. All above applications require users’ collective contribution.

In multiple cooperative task scenarios, people prefer to cooperate with trustworthy friends. Thus choosing the compatible users to perform cooperative tasks can improve the quality and success rates of mobile crowd sensing service. In addition, cooperation with compatible users is also helpful to achieve good flexibility since the compatible users can adjust the job assignment accurately and personally based on the individualities. For example, the compatible users can assign the sensing time and sensing locations according to future schedules, habits, preferences or behavior profiles [20].

In this paper, we consider that the mobile crowd sensing with multiple cooperative tasks is launched in the online community, in which the members (referred as users in the rest of this paper) are interested in participating sensing tasks. Each of cooperative tasks requires a specific amount of compatible users to perform. We explore the compatible relation via the social relation of the online community. The objective is designing truthful incentive mechanisms to minimize the social cost (the total cost of winners) such that each cooperative task can be completed by a group of compatible users. In our system model, each user submits the tasks it can perform and corresponding bid prices. Meanwhile, each user can submit a set of recommended users according to its preference. Specifically, if there is no recommended user, the user can simply submit the empty set. The platform selects a subset of users and notifies winners of the determination. The winners perform the sensing tasks and send data back to
the platform. Finally, each user obtains the payment, which is decided by the platform. The process is illustrated by Fig. 1.

![Fig. 1 Mobile crowd sensing process with multiple cooperative tasks](image)

The problem of designing truthful incentive mechanisms to minimize the social cost for such mobile crowd sensing systems is very challenging. First, the compatibility models should be defined to measure the different compatibility levels. Second, when selecting winners for tasks, the incentive mechanisms should consider not only the social optimization but also the compatibility of the users. Moreover, the user can take a strategic behavior by submitting dishonest recommended users or bid price to maximize its utility.

The main contributions of this paper are as follows:

- To the best of our knowledge, this is the first work to design truthful incentive mechanisms for the mobile crowd sensing system, where each task needs to be performed by a group of compatible users.
- We present two bid models, and formulate the Social Optimization Compatible User Selection (SOCUS) problem for each. We further present three compatibility models, which can depict the different compatibility levels, and explore the compatible relation via the social relation of online community.
- We design two incentive mechanisms MCT-M and MCT-S for two bid models. We show that the designed mechanisms satisfy desirable properties of computational efficiency, individual rationality and truthfulness. In addition, MCT-M can output the optimal solution.

The rest of the paper is organized as follows. Section II formulates two bid models and three compatibility models, and lists some desirable properties. Section III and Section IV present the detailed design of our incentive mechanisms for two bid models, respectively. Performance evaluation is presented in Section V. We review the state-of-art research in Section VI, and conclude this paper in Section VII.

II. System Model And Desirable Properties

In this section, we model the mobile crowd sensing system as a reverse auction and present two different bid models: multi-bid model and single-bid model. In the multi-bid model, each user can submit multiple task-bid pairs and can be recruited to work on a portion of submitted tasks. The single-bid model allows each user to bid single price for multiple tasks it can perform. Each user is required to perform all submitted tasks once he is selected as a winner in the single-bid model. Thus the single-bid model is suitable for the single-minded users, while multi-bid model provides more flexibility to the users. Moreover, we present three compatibility models of users: weak compatibility model, medium compatibility model and strong compatibility model. At the end of this section, we present some desirable properties.

A. Multi-bid Model

We consider a mobile crowd sensing system consisting of a social network application platform and an online community with many smartphone users. The platform resides in the cloud. The platform publicizes a set \( T = \{t_1, t_2, ..., t_m\} \) of \( m \) cooperative tasks in an online community \( U = \{1, 2, ..., n\} \) of \( n \) smartphone users, who are interested in participating sensing tasks. Each task \( t_j \in T \) is associated with the cooperative index \( \gamma_j \), which is the least amount of compatible users to perform \( t_j \).

Each user \( i \) submits a 2-tuple \( B_i = (\beta_i, \xi_i) \), where \( \beta_i = \{\beta^1_i, \beta^2_i, ..., \beta^k_i\} \) is a set of \( k \) task-bid pairs. The task-bid pair \( i \) is denoted by \( \beta^l_i = (t^l_i, b^l_i) \), \( t^l_i \in T \). Each \( b^l_i \) is associated with the cost \( c^l_i \), which is the private information and known only to user \( i \). \( b^l_i \) is the claimed cost, which is the bid price that user \( i \) wants to charge for performing \( t^l_i \). Each user can submit a set of recommended users, called compatible user set, according to its preference. The user prefers to cooperate with the users in its compatible user set to perform the tasks. We also consider that the real compatible user set is the private information and known only to user \( i \). \( \xi_i \subseteq U \) is the claimed compatible user set of \( i \).

Given the task set \( T \) and the bid profile \( B = (B_1, B_2, ..., B_n) \), the platform calculates the winning task-bid pair set \( \beta_S \subseteq \bigcup_{i \in U} \beta_i \) and the payment profile \( p^l_i \) for each winning task-bid pair \( \beta^l_i \in \beta_S \). The payment for each winner \( i \) is \( p_i = \sum_{\beta^l_i \in \beta_i \cap \beta_S} c^l_i \). A user \( i \) is called a winner and be added into winner set \( S \) if it has at least one winning task-bid pair, i.e., \( \beta_i \cap \beta_S \neq \emptyset \). We define the utility of user \( i \) as the difference between the payment and its real cost:

\[
 u_i = p_i - \sum_{\beta^l_i \in \beta_i \cap \beta_S} c^l_i \tag{1}
\]

Since we consider the users are selfish and rational individuals, each user can behave strategically by submitting a dishonest compatible user set or dishonest bid prices to maximize its utility. We assume that the truthfulness of submitted task can be achieved since they can be verified by the platform. In order to prevent the monopoly and guarantee the sensing quality, we assume each cooperative task can be completed by at least two different groups of compatible users. Here, we say two groups are different if there is at least one different user between them. This assumption is reasonable for mobile crowd sensing systems as made in [7, 8, 9]. If a task can only be completed by the unique group of compatible users, the platform can simply remove it from \( T \).

The incentive mechanism \( M(T, B) \) outputs a winning task-bid pair set \( \beta_S \) and a payment profile \( p = (p_1, p_2, ..., p_n) \). The objective is minimizing the social cost such that each of cooperative tasks in \( T \) can be completed by a group of compatible users. We will present the compatibility models in Section II-C. We refer this problem as Social Optimization Compatible User Selection (SOCUS) problem.
B. Single-bid Model

The definitions of $T, U, \zeta_i, t_i, r_j, t^t_i$ are the same as those in Section II-A. Each user $i$ submits a 3-tuple $B_i = (b_i, b_i, \zeta_i)$, where $b_i = \{t^t_i, t^t_2, ..., t^t_k\}$ is a set of $k$ tasks. The task set $b_i$ is associated with the cost $c_i$, which is the private information and known only to user $i$. $b_i$ is the claimed cost. We also consider the real compatible user set is the private information and known only to user $i$.

Given the task set $T$ and the bid profile $B = (B_1, B_2, ..., B_n)$, the platform calculates the winner set $S \subseteq U$ and the payment $p_i$ for each winner $i \in S$. We define the utility of user $i$ as:

$$u_i = p_i - c_i \quad (2)$$

A user can behave strategically by submitting a dishonest compatible user set or a dishonest bid price to maximize its utility. The incentive mechanism $\mathcal{M}(T, B)$ outputs a winner set $S$ and a payment profile $p = (p_1, p_2, ..., p_n)$. The objective is minimizing the social cost such that each of the cooperative tasks in $T$ can be completed by a group of compatible users.

C. Compatibility Model

In this subsection, we present three compatibility models to depict the different compatibility levels:

- **Weak Compatibility Model**: The two users $i$ and $j$ satisfy the weak compatibility (denote as $i \approx j$) if $j \in \zeta_i$ or $i \in \zeta_j$ for any $i, j \in U$. We consider that the relation of weak compatibility is symmetric and transitive. Then we define **Weak Compatibility Group (WCG)** as \(\{i| i \equiv j, \forall i, j \in U\}\). Essentially, the weak compatibility model is established on the one-way preferences between the users.

- **Medium Compatibility Model**: We define the transitive relation $\triangleright$: If $k \in \zeta_i \land j \in \zeta_k \land \forall i, j, k \in U$, we say $i \triangleright j$. The two users $i$ and $j$ satisfy the medium compatibility (denote as $i \triangleleft j$) if $i \triangleright j$ and $j \triangleright i$ for any $i, j \in U$. Then we define **Medium Compatibility Group (MCG)** as \(\{i| i \triangleleft j, \forall i, j \in U\}\). The medium compatibility model is established on the two-way preferences between the users.

- **Strong Compatibility Model**: The two users $i$ and $j$ satisfy the strong compatibility (denote as $i \equiv j$) if $j \in \zeta_i$ and $i \in \zeta_j$ for any $i, j \in U$. We consider that the relation of strong compatibility is symmetric and transitive. Then we define **Strong Compatibility Group (SCG)** as \(\{i| i \equiv j, \forall i, j \in U\}\). The strong compatibility model is established on the two-way preferences between the users.

Obviously, the medium compatibility model is a special case of strong compatibility model, and the weak compatibility model is a special case of medium compatibility model. We give three simple examples for illustrating $WCG$, $MCG$ and $SCG$ in Fig.2.

D. Desirable Properties

Our objective is to design the incentive mechanisms satisfying the following four desirable properties:

- **Computational Efficiency**: An incentive mechanism $\mathcal{M}$ is computationally efficient if the outcome can be computed in polynomial time.

- **Individual Rationality**: Each user will have a non-negative utility when bidding its true cost and compatible user set, i.e., $u_i \geq 0, \forall i \in U$.

- **Truthfulness**: An incentive mechanism is compatibility- and cost-truthful (called truthful simply) if reporting the true compatible user set and cost is a weakly dominant strategy for all users. In other words, no user can improve its utility by submitting a false compatible user set or cost, no matter what others submit.

- **Social Optimization**: A mechanism achieves social optimization if it can output the optimal solution of $SOCUS$ problem.

III. INCENTIVE MECHANISM FOR THE MULTI-BID MODEL

In this section, we present an incentive mechanism for Multiple Cooperative Tasks in the Multi-bid model (MCT-M). MCT-M consists of two steps: cooperative user grouping and reverse auction. MCT-M first divides the users into compatible user groups, in which each user is compatible with others. Afterwards, MCT-M performs a reverse auction mechanism to select the winning task-bid pairs and determine the payment for each user.

A. Compatible User Grouping

MCT-M first divides the users into compatible user groups based on the compatibility models defined in Section II-C, and constructs $WCG$s, $MCG$s or $SCG$s.

For the weak compatibility model, we construct an undirected graph to represent the user compatibility relation based on the claimed compatible user set. For any $i, j \in U, i \not\equiv j$, if there is $j \in \zeta_i$ or $i \in \zeta_j$, we add an edge between $i$ and $j$. Then the $WCG$s can be constructed via computing the connected components of the graph within $O(n^2)$ time.

For the medium compatibility model, we construct a directed graph. For any $i, j \in U, i \not\equiv j$, if there is $j \in \zeta_i$ or $i \in \zeta_j$, we add a directed edge from $i$ to $j$. Then we can construct $MCG$s via computing the strongly connected components of the graph, which can be solved within $O(n^2)$ time.

For the strong compatibility model, we construct an undirected graph. For arbitrary $i, j \in U, i \not\equiv j$, if there is $j \in \zeta_i$ and $i \in \zeta_j$, we add an edge between $i$ and $j$. Then we can construct $SCG$s via computing the connected components.

It is straightforward to construct the compatible user groups according to the original compatible user sets. However, the outcome of compatible user grouping depends strongly on the claimed compatible user sets. In other words, the users can change the outcome of grouping by misreporting their compatible user sets. We use the example in Fig.3 to illustrate that grouping according to the original compatible user set leads untruthfulness in weak compatibility model. Let
T = \{t_j\}, \tau_j = 2, \ U = \{1,2,3\}, \ \zeta_3 = \{2\}, \ \zeta_1 = \zeta_2 = \emptyset, \ b_1 = 1, \ b_2 = 2, \ b_3 = 3. \ All \ users \ bid \ for \ task \ j. \ We \ first \ consider \ the \ case \ where \ all \ three \ users \ submit \ real \ compatible \ user \ sets. \ Obviously, \ WCG = \{2,3\}, \ u_1 = 0 \ since \ user \ 1 \ cannot \ cooperate \ with \ any \ user. \ We \ now \ consider \ the \ case \ where \ user \ 1 \ lies \ by \ submitting \ \zeta_1 = \{2\}. \ In \ this \ case, \ WCG = \{1,2,3\}, \ S = \{1,2\} \ and \ the \ payment \ for \ user \ 1 \ would \ be \ 3 \ if \ we \ use \ VCG \ payment \ rule [10]. \ Thus, \ u_1 = 3 - 1 = 2. \ Note \ that \ user \ 1 \ improves \ its \ utility \ from \ 0 \ to \ 2 \ by \ lying \ about \ its \ compatible \ user \ set. \ The \ similar \ examples \ can \ be \ illustrated \ for \ both \ medium \ compatibility \ model \ and \ strong \ compatibility \ model.

![Fig. 3 An example showing the untruthfulness of grouping according to the compatible user sets in the weak compatibility model, where the disks represent users, and the arrows represent the compatible user sets. The numbers beside the disks represent the cost for performing task j. The dotted disks represent WCGs.](image)

**Algorithm 1: Compatible User Grouping**

```
Input: graph G
1 \ A \leftarrow \emptyset; S_{\emptyset} \leftarrow \emptyset;
2 \ foreach \ i \in \{1,2,\ldots,m\} \ do
3 \hspace{0.5cm} |U_i| \leftarrow \emptyset; A \leftarrow A \cup U_i;
4 \hspace{1cm} end
5 \ Assign \ each \ user \ independently \ and \ uniformly \ at \ random \ to \ one \ of \ m \ subsets \ U_1, U_2, \ldots, U_m;
6 \ Let \ A_{\emptyset} \subseteq A \ be \ a \ random \ subset \ with \ size \ k - m|k/m|;
7 \ foreach \ U_i \in A \ do
8 \hspace{0.5cm} if \ |U_i| \in A_{\emptyset} \ then
9 \hspace{1cm} |S_{\emptyset}| \leftarrow \emptyset; \ U_i \leftarrow \emptyset;
10 \hspace{1cm} else
11 \hspace{1.5cm} Let \ U'_i \subseteq U_i \ be \ the \ set \ of \ [k/m] \ users \ with \ highest \ indegree \ based \ only \ on \ edges \ from \ U\setminus U_i;
12 \hspace{1.5cm} |S_{\emptyset}| \leftarrow S_{\emptyset} \cup U'_i; U_i \leftarrow U_i \setminus U'_i;
13 \hspace{1cm} end
14 \hspace{1cm} else
15 \hspace{1.5cm} if \ |U_i| < [k/m] \ then
16 \hspace{2cm} |S_{\emptyset}| \leftarrow S_{\emptyset} \cup U_i; U_i \leftarrow \emptyset;
17 \hspace{2cm} else
18 \hspace{2.5cm} Let \ U'_i \subseteq U_i \ be \ the \ set \ of \ [k/m] \ users \ with \ highest \ indegree \ based \ only \ on \ edges \ from \ U\setminus U_i;
19 \hspace{2.5cm} |S_{\emptyset}| \leftarrow S_{\emptyset} \cup U'_i; U_i \leftarrow U_i \setminus U'_i;
20 \hspace{1cm} end
21 \hspace{1cm} end
22 \hspace{0.5cm} end
23 \hspace{0.5cm} end
24 \hspace{0.5cm} if \ |S_{\emptyset}| < k \ then
25 \hspace{1cm} for \ i=1 \ to \ k - |S_{\emptyset}| \ do
26 \hspace{1.5cm} Select \ j \ uniformly \ from \ U\setminus S_{\emptyset};
27 \hspace{1.5cm} |S_{\emptyset}| \leftarrow S_{\emptyset} \cup \{j\};
28 \hspace{1cm} end
29 \hspace{1cm} end
30 \ Group \ the \ users \ in \ S_{\emptyset} \ based \ on \ the \ specific \ compatibility \ model. \ Let \ \mathcal{G} \ be \ the \ set \ of \ compatible \ users \ groups.
31 \ return \ \mathcal{G};
```

To solve this issue, we introduce the Random m-Partition Mechanism (m-RP) [11], which is a randomized truthful mechanism for the approval voting [12]. We construct a directed graph G without self-loops: For any i, j \in V, i \neq j, if there is j \in \zeta_i, we add a directed edge from i to j. Then we select a subset S_k of k users maximizing the total indegree of selected users. In our system model, we adopt m-RP to select k users with the maximum recommendations. Then MCT-M constructs WCGs, MCGs or SCGs based on the recommendations of k users selected through m-RP.

The whole process of compatible user grouping is illustrated in Algorithm 1, which works as follows:

1. The users are assigned independently and uniformly at random to one of m subsets (denoted as U_1, U_2, \ldots, U_m). Let A be the set of these m subsets.
2. Select k - m\lfloor k/m \rfloor subsets from A randomly. Let A_{\emptyset} be the set of these k - m\lfloor k/m \rfloor subsets.
3. For each U_i \in A, if U_i \in A_{\emptyset}, select \lfloor k/m \rfloor users from U_i with highest indegree based on edges from U\setminus U_i; if U_i \notin A_{\emptyset}, select \lfloor k/m \rfloor users from U_i with highest indegree based on edges from U\setminus U_i.
4. If any subset U_i is smaller than the number of users needed to be selected, select all users in this subset.
5. If the size of winner set S_{\emptyset} is smaller than k, select k - |S_{\emptyset}| additional users from the unselected users uniformly.
6. Group the users in S_{\emptyset} based on the specific compatibility model.

**B. Auction mechanism design**

Consider that the outcome of compatible user grouping is a set of d compatible user groups \mathcal{G} = \{G_1, G_2, \ldots, G_d\}. Let U_{G_k} be the set of k users. MCT-M then selects a set of winners to minimize the social cost through a reverse auction such that each cooperative task can be completed by a group of users, who belong to the same compatible user group.

In the multi-bid model, each task submitted by users is with a bid price, thus we can select winning task-bid pairs for each task independently. For any task t_j \in T, we process each compatible user group G_k, k = 1,2, \ldots, d. In each iteration, we check if there are \tau_j users, who bid for t_j in G_k. If so, we select \tau_j users from G_k with minimum total bid price, and the set is denoted as S_k. Then MCT-M selects the set with minimum cost = \sum_{k=1}^{d} b_k as the winner set for t_j from all S_i, i = 1,2, \ldots, d. We apply VCG based payment rule to determine the payment for each winning task-bid pair. A winning task-bid pair will be paid an amount equal to the benefit it introduces to the system, i.e., the difference between other users’ minimum social cost with and without it:

\[ p_i = \text{cost}(U\setminus G_k \setminus \{\beta_i\}) - \text{cost}(U\setminus G_k \setminus \{\beta_i\}) - \text{cost}(U\setminus G_k), \forall \beta_i \in \beta_k \]

Here function cost() means the minimum social cost computed by MCT-M. Finally, we determine the payment for each winner i as \[ p_i = \sum_{\beta_i \in \beta_k \cap \beta_s} p_i \] . The whole process is illustrated in Algorithm 2.
Algorithm 2: Reverse Auction for Multi-bid Model

Input: task set \( T \), bid profile \( B \), compatible user group set \( G \), the set of \( k \) users \( U_g \)
// Winner Selection phase
1. \( S \leftarrow \emptyset \); \( \text{cost} \leftarrow 0 \); \( \beta_g \leftarrow \emptyset \); \( g \leftarrow \{G_1, G_2, ..., G_d\} \);
2. for \( k = 1 \) to \( d \) do \( S_k \leftarrow \emptyset \);
3. foreach \( t_j \in T \) do
   4. foreach \( k = 1 \) to \( d \) do
      5. \( S_k \leftarrow \emptyset \);
      6. \( \text{num}_{\beta_i} \leftarrow \text{argmin}_{i \in \beta_g \cap S_k \beta_i} \); \( S_k \leftarrow S_k \cup \{i\} \);
      7. until \( |S_k| \geq r_j \);
   8. end
9. end
10. \( k' \leftarrow \text{argmin}_{k \in \{1, 2, ..., d\}} \sum_{i \in \beta_g \cap S_k \beta_i} \); \( S_k \leftarrow S \cup \{k\} \);
11. foreach \( i \in S_k \) do \( \beta_i \leftarrow \beta_g \cup \{i\} \);
12. \( \text{cost} \leftarrow \text{cost} + \sum_{i \in \beta_i} \); \( S \leftarrow S \cup S_k \);
13. end
// Payment Determination Phase
14. foreach \( i \in U \) do \( p_i \leftarrow 0 \);
15. foreach \( \beta_i \in \beta_S \) do
   16. \( p_i \leftarrow \text{cost} (\text{U}_i \cup \beta_i \{1, ..., k\}) - \text{cost} (\text{U}_i \cup \beta_i \{1, ..., k\}) - b_i \);
   17. end
18. foreach \( i \in S \) do \( p_i = \sum_{\beta_i \in \beta_i \cap \beta_S} p_i \);
19. return \( \text{cost}, \beta_S, p \);

C. Mechanism Analysis

In the following, we present the theoretical analysis, demonstrating that MCT-M can achieve the desired properties.

Lemma 1. MCT-M is computationally efficient.

Proof: It suffices to prove that both Algorithm 1 and Algorithm 2 are computationally efficient.

In Algorithm 1, the running time of \( m-RP \) (Line1-29) is dominated by sorting the users in \( U_i \) (Line12 or Line19). For each of \( m \) subset, \( m-RP \) performs the sorting. The worst case happens when all users are assigned to the same subset. In this case, \( m-RP \) takes \( O(n \log n) \) time. Grouping the users in \( S_k \) (Line30) takes \( O(k^2) \) time. Thus Algorithm 1 takes \( O(max(n \log n, k^2)) \) time.

In Algorithm 2, the running time of winner selection phase is dominated by sorting the users based on bid price in each compatible user group (Line8-11). For each task in \( T \), the winner selection phase executes the sorting for each of \( d \) compatible user group. The worst case happens when all users are in the same compatible user group. In this case, the winner selection phase takes \( O(m \log k) \) time. In the payment determination phase, a process similar to winner selection phase is executed for each winning task-bid pair. Since there are at most \( \sum_{j=1}^{m} r_j \) winning task-bid pairs, running time of the Algorithm 2 is bounded by \( O((\sum_{j=1}^{m} r_j)m \log k) \).

Lemma 2. MCT-M is individually rational.

Proof: It is easy to know that MCT-M can output the optimal solution of \( \text{SOCUS} \) problem. We denote \( \text{cost}^* \) and \( \text{cost}_s^* \) as the optimal social cost of \( \text{SOCUS} \) problem in multi-bid model with and without task-bid pair \( \beta_i \), \( i \in S_j \), \( j \in T \), \( \beta_i \in \beta_g \), respectively. Then \( p_i = \text{cost}^* - \text{cost}_s^* \) based on line 21 in Algorithm 2. Since \( \text{cost}^* \) is the optimal social cost, we have \( \text{cost}^* \leq \text{cost}_s^* \), and it is easy to deduce \( p_i \geq 0 \).

Before analyzing the truthfulness of MCT-M, we firstly introduce the Theorem about \( m-RP \).

Theorem 1. (11, Theorem 4.1) For every value of \( m \), \( m-RP \) is truthfull.

The truthfulness in Theorem 1 means that no user can improve the chance of being selected into \( S_k \) by submitting a false compatible user set, no matter what others submit.

Lemma 3. MCT-M is truthfull.

Proof: The compatibility-truthfulness can be guaranteed by Theorem 1. Since we adopt VCG payment rule, which is known as a cost-truthful auction, MCT-M is cost-truthful.

The above three lemmas prove the following theorem.

Theorem 2. MCT-M is computationally efficient, individually rational, truthful and an optimal algorithm of \( \text{SOCUS} \) problem in the multi-bid model.

IV. INCENTIVE MECHANISM FOR THE SINGLE-BID MODEL

In this section, we consider the case where each user can submit a single bid price for submitted tasks, and present an incentive mechanism for Multiple Cooperative Tasks in the Single-bid model (MCT-S).

A. Mechanism Design

Similar with MCT-M, MCT-S is a two-step mechanism. The grouping method is the same as that in MCT-M. Thus we focus on solving the \( \text{SOCUS} \) problem in the single-bid model in this subsection. Unfortunately, as the following theorem shows, it is NP-hard to find the optimal solution.

Theorem 3. The \( \text{SOCUS} \) problem in the single-bid model is NP-hard.

Proof: We consider the Weighted Set Multiple Cover (WSMC) problem: there are a set \( T = \{t_1, t_2, ..., t_m\} \) of \( m \) elements, a family of sets \( \beta = \{\beta_1, \beta_2, ..., \beta_n\} \) and a positive real \( v \), each \( \beta_i \subseteq T \) has its weight \( c_i \) for \( i \in \{1, ..., m\} \), and each \( t_j \in T \) is with a positive integer \( r_j \in \{1, ..., m\} \). The question is whether exists a set \( \beta \subseteq \beta \) with \( \sum_{\beta_i \in \beta} c_i \leq v \), such that each task \( t_j \) in \( T \) is covered \( r_j \) times in the members of \( \beta \).

We can see that the \( \text{SOCUS} \) problem in the single-bid model is a generalization of the WSMC problem when each \( t_j \) only can be covered by \( r_j \) users who are within the same compatible user group. Since the WSMC problem is NP-hard, the \( \text{SOCUS} \) problem in the single-bid model is NP-hard.
Since the SOCUS problem in the single-bid model is NP-hard, we turn our attention to develop a polynomial algorithm. The main idea of MCT-S is selecting winners iteratively with minimum marginal cost for each task. Illustrated in Algorithm 3, the reverse auction still consists of the winner selection phase and the payment determination phase.

**Algorithm 3: Reverse Auction for Single-bid Model**

Input: task set $T$, bid profile $B$, compatible user group set $G$, the set of $k$ users $U_G$

// Winner Selection Phase
1. $S \leftarrow \emptyset$; $G \leftarrow \{G_1, G_2, ..., G_d\}$
2. For $k = 1$ to $d$
3. \hspace{1em} $S_k \leftarrow \emptyset$; $S_k' \leftarrow \emptyset$
4. \hspace{2em} foreach $t_j \in T$ do $cost_{j_k} \leftarrow 0$
5. \hspace{1em} end
6. \hspace{1em} foreach $t_j \in T$ in arbitrary fixed order do
7. \hspace{2em} foreach $k = 1$ to $d$
8. \hspace{3em} $S_k \leftarrow \emptyset$
9. \hspace{3em} $Q_k \leftarrow \{i | i \in S_k, t_j \in \beta_i\}$
10. \hspace{3em} num$_k \leftarrow \{|i | i \in Q_k, t_j \in \beta_i\}$
11. \hspace{3em} if num$_k \geq r_j$ then
12. \hspace{4em} if $r_j \leq |Q_k|$ then
13. \hspace{5em} break;
14. \hspace{4em} else
15. \hspace{5em} $i' \leftarrow \arg\min_{i \in Q_k \setminus \{S_k \cup \{i\}\}, t_j \in \beta_i} b_i$
16. \hspace{5em} $S_k \leftarrow S_k \cup \{i\}$
17. \hspace{5em} $cost_{j_k} \leftarrow cost_{j_k} + b_i$
18. \hspace{4em} until $|Q_k| + |S_k| \geq r_j$
19. \hspace{3em} end
20. \hspace{2em} else
21. \hspace{3em} cost$_{j_k} \leftarrow \infty$
22. \hspace{2em} end
23. \hspace{1em} end
24. \hspace{1em} $k' \leftarrow \arg\min_{k \in \{1, 2, ..., d\}} cost_{j_k}$
25. \hspace{1em} $S_k' \leftarrow S_k \cup S_{k'}$
26. \hspace{1em} $S \leftarrow S \cup S_{k'}$
27. \end

//Payment Determination Phase
28. \foreach $i \in U$ do $p_i \leftarrow 0$
29. \foreach $i \in S$ do
30. \hspace{1em} foreach $t_j \in T$ in the same fixed order do
31. \hspace{2em} Select winners from $U_G \setminus \{i\}$ for $t_j$
32. \hspace{2em} Let $cost(U_G \setminus \{i\})^{t_j}$ be the marginal cost for performing $t_j$ without $i$.
33. \hspace{2em} Let $cost(U_G)^{t_j}$ be the marginal cost for performing $t_j$ with $i$.
34. \hspace{2em} if $cost(U_G)^{t_j} < cost(U_G \setminus \{i\})^{t_j}$ then
35. \hspace{3em} $p_i \leftarrow \max\{p_i, cost(U_G \setminus \{i\})^{t_j} - (cost(U_G)^{t_j} - b_j)\}$
36. \hspace{2em} end
37. \hspace{1em} end
38. \hspace{1em} end
39. return $(S, p)$

In the winner selection phase, MCT-S processes tasks in arbitrary fixed order. For each task $t_j$, we process each compatible user group $G_k$, $k = 1, 2, ..., d$, iteratively. In each iteration, let $S_k$ be the set of winners in $G_k$ in current state, let $Q_k \subseteq S_k$ be the set of winners, who bid for $t_j$. Then MCT-S checks if there are $r_j$ users, who bid for $t_j$ in $G_k$. If so we select additional $r_j - |Q_k|$ users in $G_k$ as winners, denoted by $S_k'$, with minimum marginal cost. The minimum marginal cost of $G_k$ for $t_j$ is denoted as $cost^{t_j}_{G_k} = \min\Sigma_{i \in S_k'} b_i$. For task $t_j$, MCT-S selects $S_k'$ as the additional winner set with minimum $cost^{t_j}_{G_k}$ among all $k = 1, 2, ..., d$. The winner selection phase terminates when all tasks have been processed.

In the payment determination phase, for each winner $i \in S$, MCT-S calls the winner selection phase to select winners from $U_G \setminus \{i\}$ for all tasks iteratively. Let $cost(U_G \setminus \{i\})^{t_j}$ be the marginal cost for performing $t_j$ without $i$. If $cost(U_G)^{t_j}$ is the marginal cost for performing $t_j$ with $i$. If $i$ is a winner for $t_j$, i.e., $cost(U_G)^{t_j} < cost(U_G \setminus \{i\})^{t_j}$, we compute the maximum price of $i$ to make the group including $i$ can be selected instead of another group without $i$. We will prove that this price is a critical payment for user $i$ later.

### B. A Walk-Through Example

We use the example in Fig.4 to illustrate how the reverse auction of MCT-S works.

![Fig. 4 An example illustrating how the reverse auction of MCT-S works, where the disks represent users, the squares represent tasks. The numbers below the disks represent the costs. The numbers above squares represent compatible user groups $G_1 = \{1, 2, 3, 4\}, G_2 = \{5, 6, 7\}$. The dotted squares represent compatible user groups $G_1 = \{1, 3\}, G_2 = \{5, 6, 7\}$.](image)

**Winner Selection:**
- For task 1, $S = \emptyset$, $S'_1 = \{1, 3\}$, $cost_1^1 = b_1 + b_3 = 10$, $S'_2 = \{5, 6\}$, $cost_2^1 = b_5 + b_6 = 17$.
- For task 2, $S = \{1, 3\}$, $S'_1 = \{2, 4\}$, $cost_2^1 = b_2 + b_4 = 8$, $S'_2 = \{5, 6, 7\}$, $cost_2^2 = b_5 + b_6 + b_7 = 21$.
- For task 3, $S = \{1, 2, 3, 4\}$, $S'_1 = \emptyset$, $cost_3^1 = 0$, $S'_2 = \{5, 7\}$, $cost_3^2 = b_5 + b_7 = 13$.

During the payment determination phase, we directly give the winners when user $i$ is excluded from the consideration, due to the space limitations.

**Payment Determination:**
- $p_1$: For task 1, winners are $\{5, 6\}$, $p_1 = cost((5, 6), 1) - cost((1, 3), 1) - b_1 = 10$. For task 2, winners are $\{7\}$, $cost((1, 2, 4), 2) > cost((\{7\}))^2$. For task 3, winners are $\emptyset$, $cost((2, 3), 3) > cost((\emptyset))^3 = 0$. Thus $p_1 = 10$.
- $p_2$: For task 1, winners are $\{1, 3\}$, $cost((1, 3), 1) = cost((1, 3))^1$. For task 2, winners are $\{5, 6, 7\}$, $p_2 = cost((5, 6, 7), 2) - cost((2, 4), 2) - b_2 = 19$. For task 3,
winners are $\emptyset$, $\operatorname{cost}([2])^3 > \operatorname{cost}(\emptyset)^3 = 0$. Thus $p_2 = 19$.
- $p_3$: For task 1, winners are $\{5,6\}$, $p_3 = \operatorname{cost}([5,6])^3 - \operatorname{cost}([1,3])^3 - b_3 = 14$. For task 2, winners are $\{7\}$, $\operatorname{cost}(1,2,4)^2 > \operatorname{cost}(7)^2$. For task 3, winners are $\emptyset$, $\operatorname{cost}(2,3)^3 > \operatorname{cost}(\emptyset)^3 = 0$. Thus $p_3 = 14$.
- $p_4$: For task 1, winners are $\{1,3\}$, $\operatorname{cost}([1,3])^3 = \operatorname{cost}([1,3])^3$. For task 2, winners are $\{5,6,7\}$, $p_4 = \operatorname{cost}([5,6,7])^3 - \operatorname{cost}(12,4)^3 - b_4 = 15$. For task 3, winners are $\emptyset$, $\operatorname{cost}(2,1)^3 > \operatorname{cost}(\emptyset)^3 = 0$. Thus $p_4 = 15$.

C. Mechanism Analysis

In the following, we present the theoretical analysis, demonstrating that MCT-S can achieve the desired properties.

**Lemma 4.** MCT-S is computationally efficient.

**Proof:** Since MCT-S adopts the same compatible user grouping method (Algorithm 1) of MCT-M, the first step of MCT-S takes $O(\max(n \log n, k^2))$ time. In reverse auction step (Algorithm 3), the winner selection phase in the worst case takes $O(m k \log k)$ time, which is same as that in MCT-M. In the payment determination phase, a process similar to winner selection phase is executed for each winner. Since there are at most $\min(\sum_{j=1}^m \tau_j, k)$ winners, the running time of Algorithm 3 is $O(\min(\sum_{j=1}^m \tau_j, k) m k \log k)$. ■

**Lemma 5.** MCT-S is individually rational.

**Proof:** We assume user $i$ is selected for task $j$ in winner selection phase. Since the payment determination phase processes all tasks and compatible user groups in the same order, the output of winner selection before task $j$ would not be changed. This means that, in the payment determination phase, for task $j$, it will obtain less or equal marginal cost to choose a group of additional winners including $i$ than another group of additional winners without $i$, i.e., $\operatorname{cost}(U_g \setminus \{i\})^j < \operatorname{cost}(U_g \setminus \{i\})^j$. Hence we have $b_i < \operatorname{cost}(U_g \setminus \{i\})^j - (\operatorname{cost}(U_g)^j - b_i) \leq p_i$. This is sufficient to guarantee $b_i < \max_{j \in T}(\operatorname{cost}(U_g \setminus \{i\})^j - (\operatorname{cost}(U_g)^j - b_i)) = p_i$. ■

Before analyzing the truthfulness of MCT-S, we firstly introduce the Myerson’s Theorem [13].

**Theorem 4.** ([14, Theorem 2.1]) An auction mechanism is truthful if and only if:

- The selection rule is monotone: If user $i$ wins the auction by bidding $b_i$, it also wins by bidding $b_i' \leq b_i$.
- Each winner is paid the critical value: User $i$ would not win the auction if it bids higher than this value.

**Lemma 6.** MCT-S is truthful.

**Proof:** The compatibility-truthfulness can be guaranteed by Theorem 1. Based on Theorem 4, it suffices to prove that the selection rule of MCT-S is monotone and the payment $p_i$ for each $i$ is the critical value. The monotonicity of the selection rule is obvious as bidding a smaller value cannot push user $i$ backwards in the sorting.

We next show that $p_i$ is the critical value for $i$ in the sense that bidding higher $p_i$ could prevent $i$ from winning the auction. Note that in the iteration of $t_j$, $p_i = \operatorname{cost}(U_g \setminus \{i\})^j - (\operatorname{cost}(U_g)^j - b_i)$. If user $i$ bids $b_i > p_i$, the group of additional winners including $i$ would be replaced by another group without $i$ since $b_i > \operatorname{cost}(U_g \setminus \{i\})^j - (\operatorname{cost}(U_g)^j - b_i)$ implies $\operatorname{cost}(U_g)^j > \operatorname{cost}(U_g \setminus \{i\})^j$. Hence, user $i$ would not win the auction for $t_j$. Based on line 36 in Algorithm 3, $p_i = \max_{j \in T}(\operatorname{cost}(U_g \setminus \{i\})^j - (\operatorname{cost}(U_g)^j - b_i))$. User $i$ would not win the auction because each $t_j \in T$ has chosen a group of additional winners without $i$.

The above three lemmas prove the following theorem.

**Theorem 5.** MCT-S is computationally efficient, individually rational and truthful in the single-bid model.

V. PERFORMANCE EVALUATION

We have conducted thorough simulations to investigate the performance of MCT-M and MCT-S for all three compatibility models. Due to the space limitations, we only give the numerical results under weak compatibility model. To investigate the performance of social optimization for SOCUS problem, we also implement two benchmark algorithms without considering the compatibility among users: Benchmark-M for multi-bid model and Benchmark-S for single-bid model, respectively. Specifically, there is no compatible user grouping step in both benchmark algorithms. In Benchmark-M, a set of winning task-bid pairs with minimum social cost is selected from original user set $U$ for each cooperative task. The objective of Benchmark-S is selecting a set of winners with minimum social cost for the multiple cooperative tasks. Obviously, this is equivalent to the weighted set multiple cover problem described in the proof of Theorem 3. It is known that the greedy based approximation algorithm [22] can approximate the optimal solution within a factor of $\ln(n) + 1$.

We measure the number of winners, social cost, running time and overpayment ratio (a metric to measure the frugality of a mechanism [23], calculated by $\frac{\sum_{i \in U} \operatorname{cost}(S) - \operatorname{cost}(S)}{\operatorname{cost}(S)}$, and reveal the impacts of the key parameters, including the number of users ($n$), the number of cooperative tasks ($m$) and cooperative index ($r$).

A. Simulation Setup

The simulations are based on Wikipedia vote network [21], which contains all the Wikipedia voting data for admittance election from the inception of Wikipedia till January 2008. Nodes in the network represent Wikipedia users and a directed edge from node $i$ to node $j$ represents that user $i$ voted on user $j$. There are 7115 nodes and 103689 edges in the network.

For our simulations, we select a set of users uniformly from whole Wikipedia vote network, and construct a sub-network only consisting of selected users and the edges among them. We set the compatible user set of arbitrary user as the set of users it voted on within the sub-network. We set the default value of parameters as follows: The cost of each bid is uniformly distributed in $[5, 10]$. The cooperative index and the number of bidding tasks of each user are uniformly distributed in $[2, 5]$ and $[3, 5]$, respectively. $n=300$, $m=10$, $k=250$. However, we will vary the value of key parameters for
exploring the impacts of these parameters respectively. All the simulations were run on a Ubuntu 14.04.4 LTS machine with Intel Xeon CPU E5-2420 and 16 GB memory. Each measurement is averaged over 100 instances.

![Image](47x624 to 174x713)

**Fig. 5 Impact of the number of users**

(a) Number of groups
(b) Social cost
(c) Running time
(d) Overpayment ratio

![Image](176x624 to 303x617)

**Fig. 6 Impact of the number of cooperative tasks**

(a) Number of winners
(b) Social cost
(c) Running time
(d) Overpayment ratio

![Image](305x624 to 433x411)

**Fig. 7 Impact of cooperative index**

(a) Number of winners
(b) Social cost
(c) Running time
(d) Overpayment ratio

B. Impact of $n$

To investigate the scalability of designed mechanisms, we vary the number of users from 300 to 900, and select 80% users for each instance through $m$-RP, i.e., $k = 0.8 \times n$. As shown in Fig.5, the number of compatible user groups goes up under all three compatibility models when the number of users increases. There are 2.5, 1.75 and 1.32 users in each WCG, MCG and SCG on average, respectively. The social cost decreases with increasing user number since the platform can find more cheap users. However, the change of social cost is very slight because in our system model, the user number needs to be large enough in order to complete all cooperative tasks. The social cost of MCT-M is very close to that of BenchmarkM (only 1.8% more social cost than BenchmarkM on average) since MCT-M can output optimal solution in the multi-bid model. However, MCT-S outputs 48.9% more social cost than BenchmarkS on average. Moreover, the designed mechanisms are computational efficient since the running time of MCT-M and MCT-S is bounded by 0.8s and 0.4s, respectively, even there are 900 users. Based on the frugality theory, the overpayment ratio depends on the competition among users. As seen from Fig.5(d), the overpayment ratio of both MCT-M and MCT-S decrease because the competition among users intensify when there are more users. The overpayment ratio of MCT-M is less than that of MCT-S. Obviously, the competition of users in MCT-M is more than that of MCT-S since MCT-M can select winning task-bid pairs independently from all task-bid pairs and the cost of each task follows the identical distribution.

C. Impact of $m$

The number of cooperative tasks can depict the workload of mobile crowd sensing system. We fix $n = 300$, $k = 250$, and vary $m$ from 6 to 14. As shown in Fig.6, the number of winners and the social cost increase severely in both MCT-M and MCT-S with increasing $m$ since the platform needs more users to complete the tasks. The winners of MCT-M are much more than that of MCT-S because any user will be the winner if one of the task-bid pairs it submits is selected in the multi-bid model. Accordingly, the social cost of MCT-M is more than that of MCT-S since the cost of each winner follows the identical distribution in our settings. The running time also increases with increasing tasks. However the running time of MCT-M is still lower than 0.4s when there are 300 users and 14 cooperative tasks. The overpayment ratio also increases since the platform needs to recruit more users to perform tasks, which mitigates the competition among users accordingly.

D. Impact of $r$

To investigate the performance for the tasks associated with different cooperative levels, we vary the distribution interval of cooperative index from [2, 2] to [2, 8]. As can be seen from Fig.7, MCT-S cannot output the solution when the cooperative index is too large (the upper limit of distribution interval exceeds 7). Both the winners and the social cost
increase with increasing cooperative level since the platform needs more users to perform each cooperative task averagely. $MCT-M$ and $MCT-S$ output 6.7% and 52.6% more social cost than benchmark algorithms, respectively. The running time and overpayment ratio also increase when the cooperative index goes up. The running time of $MCT-S$ is only 32.9% of that of $MCT-M$, while the overpayment of $MCT-M$ is much less than that of $MCT-S$.

VI. RELATED WORK

Many incentive mechanisms for mobile crowd sensing have been proposed thus far. Yang et al. proposed two different models for smartphone crowd sensing [9]: the platform-centric model where the platform provides a reward shared by participating users, and the user-centric model where users have more control over the payment they will receive. In [7], Feng et al. formulated the location-aware collaborative sensing problem as the winning bids determination problem, and presented a truthful auction using the proportional share allocation rule proposed in [15]. Koutsopoulos designed an optimal reverse auction [14], considering the data quality as user participation level. However, the quality indicator, which essentially measures the relevance or usefulness of information, is empirical and relies on user’s historical information. In [16], Zhao et al. investigated the online crowdsourcing scenario where the users submit their profiles to the crowdsourcer when they arrive. The objective is selecting a subset of users for maximizing the value of the crowdsourcer under the budget constraint. They designed two online mechanisms, $OMZ$, $OMG$ for different user models. Zhang et al. proposed IMC [17], which consider the competition among the requesters in crowdsourcing. However, all above works focus on the multiple independent task scenarios, where each task only needs one user to perform.

Some works aim to the single cooperative task scenario, where the task requires a group of users to perform cooperatively. Xu et al. proposed truthful incentive mechanisms for the mobile crowd sensing system where the cooperative task is time window dependent, and the platform has strong requirement of data integrity [8]. Furthermore, they studied the budget feasible mechanisms for the same crowd sensing system [20]. Luo et al. designed the truthful mechanisms for multiple cooperative tasks [19, 24]. However, they don’t consider the compatibility among users.

Overall, there is no off-the-shelf incentive mechanism designed in the literature for the mobile crowd sensing system, where there are multiple cooperative tasks, and each of tasks requires a group of compatible users to perform.

VII. CONCLUSION

In this paper, we have designed the incentive mechanisms for the mobile crowd sensing system with multiple cooperative tasks. We explored the compatible relations via the social relations of the online communities. We have presented two bid models and three compatibility models for this new scenario, and designed two incentive mechanisms: $MCT-M$ and $MCT-S$ to solve the $SOCUS$ problem for the two bid models, respectively. Through both rigorous theoretical analyses and extensive simulations, we have demonstrated that the proposed incentive mechanisms achieve computational efficiency, individual rationality and truthfulness. Moreover, $MCT-M$ can output the optimal solution.

REFERENCES