Abstract—Crowdsensing is a sensing method which involves participants from general public to collect sensed data from their mobile devices, and also contribute and utilize a common database. To ensure a crowdsensing system to operate properly, there must be certain effective and efficient incentive mechanism to attract users and stimulate them to submit sensing data with high quality. Intuitively, the agreement on the qualities and payments in crowdsensing systems can be best modeled as a contract. However, none of existing incentive mechanisms consider data quality through effective contract design. In this paper, we design two quality-aware contract-based incentive mechanisms for crowdsensing, named QUAC-F and QUAC-I, under full information model and incomplete information model, respectively, which differ in the level of users’ information known to the system. Both QUAC-F and QUAC-I are guaranteed to maximize the platform utility while satisfying individual rationality and incentive compatibility. We evaluate the performance of our designed mechanisms based on a real dataset.

I. INTRODUCTION

The term “crowdsensing” refers to acquiring sensor data from a large and diffuse group of mobile devices and maintaining and sharing the knowledge by a common database. The sensing devices may include, for example, smartphones, sensor embedded gaming systems, and in-vehicle sensor devices. Due to the rapid emergence of these mobile devices and their remarkable improvement of sensing capability and analyzing techniques, crowdsensing is receiving more and more attention, not only from industry, but also academia.

A typical crowdsensing system consists of three main components, the platform, the requesters and the users. The requesters provide tasks they are willing to pay people to do. The platform then publishes the tasks and the payment rules. The users can choose the tasks that they are interested in and capable of performing. At last, after completing their selected tasks and submitting the data, the users will be paid accordingly. In our model, we only consider the interaction between the platform and the users. Without loss of generality, we assume that the platform makes the decision on the payment rules.

Compared to traditional business models, the platform can save significant financial resources since they no longer need to hire experts or deploy special devices. However, such advantages are valid only if there are enough users participating in the crowdsensing system and they are providing sensing data with good quality. To incentivize the users, the platform either provides services to or pays the users with money in return. The question is “how much should the users be awarded?”

There exist some auction-based incentive mechanisms which consider both the payment and the quality. Their main approach is to choose the users who ask less money and provide higher quality. However, auction-based mechanisms suffers from the long period of time before the getting the auction result, and has a higher risk of not recruiting enough users. Moreover, no existing auction-based mechanisms considering the fact that users have the ability to change the qualities of the sensing data. In addition, auction-based mechanisms may be hard to implement [1] and result in price discrimination [2].

Therefore, we choose to use contract-based incentive mechanisms over auctions. The platform will publish quality-payment bundles to users and let them make their decisions to maximize their utilities and perform tasks with corresponding qualities. By designing the contracts carefully, we aim to maximize the utility of the platform. Note that contracts are the most intuitive way to define the relationship between the quality and the payment and have already been used in many crowdsourcing/crowdsensing platforms, such as Amazon Mechanical Turk and Gigwalk.

In this paper, we consider two models, full information model and incomplete information model. In the full information model, the platform has the knowledge of users’ basic characteristics, e.g., ability distribution, cost function and risk attitude, but does not know how much effort the users will exert to complete tasks. The platform determines a contract that specifies the expected quality and a payment function which is a function of data quality. Given the contract, the users decide how much effort to exert in order to maximize their utilities. In the incomplete information model, the basic characteristic information is private to the users themselves. But the platform knows the distribution of users’ characteristics and designs a contract consisting of multiple contract items, which are the desired qualities and their corresponding payments. Given the contract, the users choose and fulfill the contract items that maximize their utilities. For each of these two models, we design a QUality-Aware Contract-based incentive mechanism, named QUAC-F and QUAC-I, respectively, which maximizes the platform utility while guaranteeing individual rationality and incentive compatibility.

Note that since crowdsensing system can be regarded as a special crowdsourcing system, our mechanisms are also applicable to crowdsourcing system where the quality can be quantified for submitted tasks.

The contributions of our work are:

1) To the best of our knowledge, we are the first to design quality-aware contract-based incentive mechanisms for both full information model and incomplete information
model where the platform knows different levels of users’ information. Such mechanisms represent reality closely. 

2) In the full information model, we allow uncertainty in the intended qualities of users’ submitted data with their efforts. We also consider that users’ different attitudes toward risk affect their choices. We design a quality-aware contract-based incentive mechanism, named QUAC-F, which maximizes the platform’s utility while guaranteeing individual rationality and incentive compatibility. QUAC-F also guarantees that users will exert extra efforts to increase data qualities.

3) In the incomplete information model, we consider possible errors of the platform evaluating the data qualities. We design a quality-aware contract-based incentive mechanism, named QUAC-I, which maximizes the platform’s utility while guaranteeing individual rationality and incentive compatibility.

II. RELATED WORK

A. Existing Incentive Mechanisms for Crowdsensing

There is a large body of research on the design of incentive mechanisms for crowdsourcing [3–14]. The above papers served the purpose to recruit users effectively, but could not incentivize high quality. Note that crowdsensing can be regarded as a special type of crowdourcing. Thus many incentive mechanisms for crowdsourcing can be applied to crowdsensing. Therefore, we also include them in this section.

There are also a few papers considering quality in the design of incentive mechanisms for crowdourcing. Gao et al. [15] proposed a cost-effective mechanism that employs quality-aware worker training as a tool to stimulate workers to provide high-quality solutions. However, this mechanism assumed users choose actions to optimize their long-term utilities, thus it is not applicable to crowdsourcing systems where users join and leave dynamically. Through randomized behavioral experiments on Amazon Mechanical Turk, Ho et al. [16] showed that simple performance-based payment schemes with fixed bonus may not always incentivize high quality data. Peng et al. [17] applied the expectation maximization algorithm to estimate quality and information theory to quantify user contribution. They then determined the payment proportional to user contribution. However, their mechanism maximizes the platform utility, guarantees individual rationality, but fails to satisfy incentive compatibility. Han et al. [18] studied a quality-aware Bayesian pricing problem and design algorithms to choose a single payment to recruit enough users with reasonable sensing qualities while minimizing the total payment.

In all, current incentive mechanisms for crowdsensing can be classified to two groups, pricing-based mechanism and auction-based mechanism. Pricing-based mechanisms are used when the platform knows how users evaluate their efforts so that they will perform the task once given the payment. However, the valuation information is hardly known to the platform in reality. Auction-based mechanisms are used when the valuation of the users’ efforts are unknown so that the platform can hold auctions to reveal the expecting payment. Such mechanisms can make the tasks performed with the lowest costs, but can not guarantee the quality of sensing results. Note that Jin et al. [19] tried to consider the data quality when deciding the winners of auction. However, their technique can not incentivize users to improve the data quality, since the truthfulness property only holds on the money they requested, but not the qualities they can provide.

In this paper, we propose two quality-aware contract-based incentive mechanisms based on different level of knowledge on users’ characteristics. Such mechanisms represent reality closely but have never been studied before, especially when considering the risk attitude and evaluation errors.

B. Contract-Based Mechanisms in Other Applications

Although contract-based incentive mechanism is new to the crowdsensing paradigm, it has been applied in many other applications. Knapper et al. [20] applied contract theory to cloud computing marketplaces to optimize the service provider’s profit. Gao et al. [21] modeled the spectrum trading in cognitive radio networks as a monopoly market and designed a feasible monopolist-dominated quality-price contract to maximize the utility of the primary user. Zhang et al. [22] proposed a contract-theoretic approach for device-to-device communications in cellular networks to motivate user involvement. However, we can not directly apply the above mechanisms to crowdsensing since they are not designed to incentivize high quality data. In addition, we consider the uncertainty in data quality and the imperfection of quality evaluation in our paper.

III. SYSTEM MODEL

In this paper, we consider a crowdsensing system consisting of one platform and \( n \) users. Based on different levels of information known to the platform, we consider two models, full information model and incomplete information model. In the full information model, the platform knows the basic characteristics (e.g. ability distribution, cost function and risk attitude, etc) of users based on their previous working histories, but does not know their actions (how much effort to contribute). In the incomplete information model, however, the platform only knows the distribution of users’ general characteristics which are related to users’ ability and willingness.

The contract-based incentive mechanisms under both models proceed similarly. For every task, the platform will design a contract which consists of quality-payment bundles. We call the contract maximizing the platform’s utility the optimal contract. Here the data quality may refer to resolution, contrast, and sharpness of photos, accuracy of GPS locations, and estimation accuracy of air quality. After the task and the contract are published to the users, each user will make their choices and perform the task with the prescribed quality. The users also have the right to reject all contracts. Once the task is finished, each user sends the data back to the platform. The platform will evaluate the quality of submitted data and pay the users according to the contract. The data quality can be evaluated by calculating the deviation from ground truths, if available, or using algorithms proposed in [23], otherwise. This process is shown in Fig. 1. Since all the steps are repeatedly performed for each task, we will only consider one task in the rest of the paper.
While designing contract-based incentive mechanisms, we strive to satisfy the following two properties:

Definition 1: Individual Rationality (IR): A contract-based incentive mechanism is individual-rational if any user achieves at least its reserved utility by making a choice to maximize its utility.

Definition 2: Incentive Compatibility (IC): A contract-based incentive mechanism is incentive-compatible if any user achieves maximum utility by contributing effort to provide expected quality specified in contract item corresponding to its characteristics.

These two properties are desired because of the following reasons. IR property ensures user participation. Note that the reserved utility of a user might be greater than zero if the user has other working opportunities. If IR is not satisfied, the users may not participate in the crowdsensing system. Without enough participating users, the effectiveness of the system will be significantly degraded. Therefore, we must ensure that any user can achieve its reserved utility with our incentive mechanisms\(^1\). IC property ensures the platform takes control of the system. Note that users are selfish and always try to maximize their utilities. If the platform wants to induce a user to submit data with certain quality, instead of some other (presumably lower) quality, then the incentive mechanism must set the payment for that quality so that the user achieves its maximum utility in this case. Only by controlling users’ actions through IC can the platform control the whole system.

A. Full Information Model

In this model, the users have different abilities towards performing the task, affected by education level, skill level, experience, etc. Once participating, each user can control how much effort they invest in the task, which is the increment of data quality achieved by contributing more time on the task, learning related skills, or asking friends for help, and so on. Naturally, exerting different levels of effort incurs different cost to users. We describe such disutility from effort by the cost function. Last but not least, users differ in the risk attitude, which is defined as the degree of risk aversion [24]. Generally, users are either risk-averse, risk-neutral, or risk-seeking [25].

We define the aforementioned characteristics as user types. We assume there are \(m\) types. Let \(n_i\) denote the number of type-\(i\) users. Thus, \(\sum_{i=1}^{m} n_i = n\). Users of the same type have similar characteristics. The platform knows the type of each user and thus its characteristic information. However, the platform can not directly know how much effort the users will exert in performing a task.

\(^1\)We eliminate users who contribute negative utilities to the platform due to the IR property. Details can be found in Section IV and Section V.

For a user of type-\(i\), we denote its ability by \(\eta_i\), effort by \(e_i\), and submitted data quality by \(q_i\). Then we have,

\[
q_i = \eta_i + e_i + \epsilon_i, \tag{1}
\]

where \(\epsilon_i\) is an error due to the uncertainty in the data quality even the user puts the same level of effort. We assume \(\eta_i\) and \(e_i\) are random variables following the normal distribution. In detail, \(\eta_i \sim N(\mu_i, \sigma_{\eta,i}^2)\) and \(e_i \sim N(0, \sigma_{e,i}^2)\), where \(\mu_i\) and 0 are means, and \(\sigma_{\eta,i}^2\) and \(\sigma_{e,i}^2\) are variances. As far as we know, this is the first time that such user model is introduced in crowdsensing. However, in economics, it is essential and widely used to model the worker’s output based on labor input [26, 27].

For users of type-\(i\), the platform will design a \((q_i^i, p_i)\) bundle, referred to as a contract item, where \(q_i^i\) is the quality that the platform expects, and \(p_i\) is a payment function of data quality. We define the payment function \(p_i\) as

\[
p_i = f(q_i), \tag{2}
\]

where \(f(\cdot)\) is a nonnegative and nondecreasing function. Note that here the contract item is not a simple single quality-payment bundle, but instead an expected quality with a payment function. With the presence of the payment function, the single contract item is essentially an infinite number of quality-payment bundles. Users can choose different effort levels so as to provide different data qualities and receive corresponding payments. An optimal payment function would incentivize users to provide data quality the same as the expected quality. Since the user type is known to the platform, the platform only needs to publish one contract item to each user.

The cost function for type-\(i\) users is denoted by \(c_i(e_i)\), which is assumed to be increasing, convex, and differentiable, with \(c_i'(0) = 0\), \(c_i'(\infty) = \infty\), \(c_i'' > 0\) and \(c_i''' > 0\). The utility of type-\(i\) users is defined as a mean-variance utility [28]

\[
U_i = E[u_i] - r_i V[u_i], \tag{3}
\]

where

\[
u_i = p_i - c_i(e_i), \tag{4}
\]

and \(r_i\) is the degree of risk aversion. Based on the discussion in [24], mean-variance utility functions are the best to represent utility in the economics of uncertainty.

If \(r_i > 0\), the users are risk-averse and prefer with a more certain, but possibly lower payment. If \(r_i = 0\), the users are risk-neutral. They are interested only in the expected utility and indifference to risk. If \(r_i < 0\), the users are risk-seeking and overweight high-payment but low-probability events. Since a user can only control its effort to change its utility, we also use \(U_i(e_i)\) to denote \(U_i\). For any user of type-\(i\), it will participate in the crowdsensing system only when its utility is at least its reserved utility, denoted by \(u_i\). Then it will choose the effort level which maximizes its utility.

Note that all the aforementioned user characteristics (ability distribution of \(\eta_i\), error distribution of \(\epsilon_i\), cost function \(c_i(\cdot)\) and risk attitude \(r_i\)) can be available to the platform by maintaining historical records of users [18, 19].

Now we can rigorously define the IR and IC in this model as follows: 1). A contract-based incentive mechanism is individual-rational if

\[
\max_{e_i} U_i(e_i) \geq u_i, \tag{5}
\]
for any type-$i$, $1 \leq i \leq m$. 2). A contract-based incentive mechanism is incentive-compatible if

$$U_i(e_i^*) \geq U_i(e_i), \quad (6)$$

for any $e_i \neq e_i^*$ for any type-$i$, $1 \leq i \leq m$, where $e_i^* = q_i^* - \mu_i$.

For any user of type-$i$, we define the utility gained from this user as

$$U_{P,i} = \mathbb{E}[v(q_i) - p_i], \quad (7)$$

where $v(\cdot)$ represents how the platform values the data quality and is assumed to be nonnegative, nondecreasing and concave.

Then the utility of the platform is defined as

$$U_P = \sum_{i=1}^{m} n_i U_{P,i}. \quad (8)$$

**Problem Statement:** Given $n$ users of $m$ user types with their characteristic information in the full information model, we aim to design an individual-rational and incentive-compatible contract-based incentive mechanism such that the utility of the platform is maximized.

### B. Incomplete Information Model

In this model, the users’ basic characteristic information is not known to the platform. The user type is defined as a comprehensive descriptor of all characteristics mentioned in the full information model. There are $m$ user types in this model. Each user type has only two attributes, the type value and the default quality. For any type-$i$, the type value $\theta_i$ describes user’s ability and willingness, and the default quality $q_i$ is the quality of data submitted by type-$i$ users without exerting either insufficient or extra effort. How to determine the type value $\theta_i$ is largely dependent on the sensing tasks, and is orthogonal to our research. Practical examples can be found in [20–22]. Without loss of generality, we assume $0 < \theta_1 < \theta_2 < \cdots < \theta_m$, and $0 \leq q_1 \leq q_2 \leq \cdots \leq q_m$.

Note that the types are all distinct, but the quality can be the same. Such situation happens when two users have the same ability but different levels of willingness to complete the task with the same quality. Moreover, the platform no longer has full information about users’ types but knows the user type distribution. In detail, the probability that a user belongs to type-$i$ is denoted by $\lambda_i$. Clearly, $\sum_{i=1}^{m} \lambda_i = 1$.

For users of type-$i$, the platform specifically designs a $(q_i, p_i)$ bundle, referred to as a contract item, where $q_i$ is the default quality for type-$i$ users and $p_i$ is the corresponding payment. Then we denote the contract as $(Q, P) = \{(q_i, p_i) \mid i \in \{1, 2, \ldots, m\}\}$. In this model, we assume that once a user selects a contract item, it will stick to the choice and perform accordingly; otherwise, it would choose another contract item in the first place. This assumption means a user can be dishonest about its type; but once the contract is signed, the user will fulfill the contract.

Note that it is very likely that the platform can not perfectly correctly verify the qualities of data since a perfect evaluation system might be costly, if not impossible. Thus we consider imperfect evaluation system and use $\pi_{i,j}$ to denote the probability that data quality $q_i$ is evaluated as data quality $q_j$.

In this model, we assume all users are risk-neutral. Therefore, the mean-variance utility function is simplified to an expected utility function. In detail, the utility of a type-$i$ user selecting contract item $(q_j, p_j)$ is directly defined as the difference between the gain from the expected payment and the cost from performing the task with $q_j$:

$$U_{i,j} = \theta_i \sum_{k=1}^{m} \pi_{j,k} p_k - \beta q_j, \quad (9)$$

where $\beta$ is a fixed parameter to convert the value of data quality to the cost of the user. For simplicity, we assume $\beta = 1$ in this paper. Even if $\beta$ is not 1 in practice, we can set it to be 1 by scaling type values. Note that instead of directly using the expected payment as the reward, we use the production of the type value and the expected payment. This is because for a user of higher type value, it takes less effort to complete the task with the same quality, physically and/or mentally. For example, considering two users submitting data of the same quality, the user of higher type may be happier and/or use less energy to finish the task, thus the overall utility should be larger. For any user, it will participate in the crowdsensing system only when it can achieve at least its reserved utility $u$, which is the same for all user types.

Now we can rigorously define the IR and IC in the incomplete information model as follows: 1). A contract-based incentive mechanism is individual-rational if

$$\max_{1 \leq i \leq m} U_{i,j} \geq u_i, \quad (10)$$

for any type-$i$, $1 \leq i \leq m$. 2). A contract-based incentive mechanism is incentive-compatible if

$$U_{i,i} \geq U_{i,j}, \quad (11)$$

for any type-$i$ and type-$j$, $1 \leq i, j \leq m$, $i \neq j$.

Finally, the utility of the platform is defined as the difference between the value of data qualities and the payments to users,

$$U_P = \sum_{i=1}^{m} n_i (v(q_i) - \sum_{k=1}^{m} \pi_{i,k} p_k), \quad (12)$$

where $n_i$ is the number of users selecting contract item $(q_i, p_i)$, $v(\cdot)$ is valuation function of data quality, and $\sum_{k=1}^{m} \pi_{i,k} p_k$ is the expected payment to a user submitting data of quality $q_i$.

**Problem Statement:** Given $n$ users, and user type distribution of $m$ types, $\lambda_1, \lambda_2, \ldots, \lambda_m$, in the incomplete information model, we aim to design an individual-rational and incentive-compatible contract-based incentive mechanism such that the utility of the platform is maximized.

### IV. QUAC-F: Contract-Based Incentive Mechanism for Full Information Model

In this section, we design an efficient contract-based incentive mechanism which maximizes the utility of the platform.

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1We plan to eliminate this assumption in future work.
A. Design Rationale

Note that the design of contract-based incentive mechanism for the full information model can be formulated as a Stackelberg game. In this game, the platform is the leader and the users are followers. The strategy of the platform is to set the expected quality $q_i^*$ and the payment function $p_i = f(q_i)$ in each contract item. The strategy of each user is to decide the data quality by exerting different levels of effort. Both the platform and the users are interested in maximizing their own utilities. Following the convention of analyzing Stackelberg games, we use backward induction to design the mechanism.

B. Design of QUAC-F

The general steps to design the contract for each type-$i$ is:

1) Given any payment function $f(\cdot)$, calculate the optimal effort level $e_i^*$ for each user by solving the first order condition of its mean-variance utility (3): $e_i^* = e |_{dU_i/de_i=0}$.
2) Plug $e_i^*$ in the first order condition of the platform’s utility (7), and then solve it to calculate the optimal parameters of $f(\cdot)$.
3) Repeat the previous two steps for different types of $f(\cdot)$ and choose the one that maximizes the platform’s utility (7).
4) Calculate the expected quality $q_i^*$ based on the effort level $e_i^*$ that maximizes type-$i$ users’ utilities under the chosen payment function.

To illustrate how QUAC-F works, we use $f(q_i) = a_i + b_i q_i$ and $v(q_i) = \alpha q_i$ as an example.

Calculation of Optimal Effort: Since users of different types are independent of each other, we consider each user type individually.

By (1), (2), and (4), we obtain

$$u_i = a_i + b_i \eta_i + b_i e_i - c_i(e_i) + b_i(\eta_i + \epsilon_i).$$

Thus we have,

$$E[u_i] = a_i + b_i \mu_i + b_i e_i - c_i(e_i),$$

and

$$V[u_i] = b_i^2 (\sigma_{\eta_i}^2 + \sigma_{\epsilon_i}^2).$$

Then the utility of a type-$i$ user is

$$U_i = a_i + b_i e_i - c_i(e_i) - r_i b_i^2 (\sigma_{\eta_i}^2 + \sigma_{\epsilon_i}^2).$$

Note that $U_i$ is concave since the cost function $c_i$ is convex. Then the first order condition for maximizing $U_i$ gives us

$$b_i - c_i'(e_i) = 0.$$  

Therefore, the utility of a type-$i$ user is maximized when

$$e_i = \hat{e}_i(b_i),$$

where $\hat{e}_i$ represents the inverse function of the first derivative of the cost function for type-$i$ users, i.e., $\hat{e}_i(\cdot) = (c_i'(\cdot))^{-1}(\cdot)$. This guarantees the IC property.

To further analyze how the choice of $b_i$ affects users’ actions, we differentiate (14) with respect to $b_i$ and obtain

$$1 - c''_i(\hat{e}_i(b_i)) \frac{d\hat{e}_i(b_i)}{db_i} = 0.$$  

Therefore,

$$\frac{d\hat{e}_i(b_i)}{db_i} > 0,$$  

which suggests that the platform can induce greater effort from users by increasing the share of quality in the payment function.

Calculation of Optimal Payment Function: Now we know how users will act given any contract. We continue to use backward induction to analyze the utility of the platform. Since users of different types are independent from each other, maximizing platform’s total utility (8) is equivalent to maximizing the utility (7) for each individual user type.

Combining (1) and (7), we get

$$U_{P,i} = \alpha(\mu_i + e_i) - E[p_i].$$

Note that to achieve the maximum utility, the platform will always set the value of $a_i$ to make (5) just bind and thus guarantee the IR property. Then we have

$$U_i = E[p_i] - c_i(e_i) - r_i b_i^2 (\sigma_{\eta_i}^2 + \sigma_{\epsilon_i}^2) = \eta_i.$$  

Moving terms, we get

$$E[p_i] = \eta_i + c_i(e_i) + r_i b_i^2 (\sigma_{\eta_i}^2 + \sigma_{\epsilon_i}^2).$$

Combining (17) and (18), we obtain

$$U_{P,i} = \alpha(\mu_i + e_i) - \eta_i - c_i(e_i) - r_i b_i^2 (\sigma_{\eta_i}^2 + \sigma_{\epsilon_i}^2).$$

The first-order condition for maximizing (19) is

$$(\alpha - c_i'(\hat{e}_i(b_i))) \frac{d\hat{e}_i(b_i)}{db_i} - 2r_i b_i (\sigma_{\eta_i}^2 + \sigma_{\epsilon_i}^2) = 0.$$  

The optimal value of $b_i$ is the solution to (20), denoted by $b_i^*$. Based on (15), the optimal effort for a type-$i$ user is $e_i^* = \hat{e}_i(b_i^*)$. Since $\max_{e_i} U_i(e_i) = U_i(e_i^*) = \eta_i$, the optimal value of $a_i$ is $a_i^* = \eta_i - b_i^* e_i^* - b_i^* \mu_i - c_i(e_i^*) + r_i(b_i^*)^2 (\sigma_{\eta_i}^2 + \sigma_{\epsilon_i}^2)$ based on (13). As the other component in each contract item, the expected quality is set to be $p_i^* = E[p_i] = e_i^* + \mu_i$.  

Remark: Note that, for any type-$i$, if there is no solution to (20) or the corresponding utility of the platform $U_{P,i}$ is negative, then we will remove type-$i$ users from consideration.

Impact of Risk Attitude on Users’ Efforts: From (20), we can observe that $b_i^*$ is based on the risk attitude $r_i$.

In order to analyze the impact of risk attitude on users’ efforts, we first introduce social welfare, which is defined as the total utility of both the platform and the users, i.e.,

$$W = \sum_{i=1}^{m} n_i U_i + U_P = \sum_{i=1}^{m} n_i (U_i + U_{P,i}).$$

Let $W_i$ be the social welfare contributed by each type-$i$ user, i.e., $W_i = U_i + U_{P,i}$. By (4) and (7), we have

$$W_i = \alpha \mu_i + \alpha e_i - c_i(e_i).$$

Since $\epsilon_i > 0$ and $\epsilon_i' > 0$, the first order condition for maximizing $W_i$ is $\alpha = c_i'(e_i)$. Define the social optimal effort for type-$i$ users, denoted by $e_i^{opt}$, as the effort that maximizes $W_i$. Then, $e_i^{opt} = \hat{e}_i(\alpha)$. For any type-$i$, by inducing all type-$i$ users to contribute effort $e_i^{opt}$, the social welfare is maximized. However, not all optimal contracts can induce the users to choose their social optimal efforts. In fact, from (16) and (20), we have the following conclusions:

1) For risk-averse users ($r_i > 0$), (20) holds only if $\alpha - c_i'\hat{e}_i(b_i^*) > 0$. Therefore, $b_i^* < \alpha$, and the platform can only induce the users to contribute efforts lower than $e_i^{opt}$.
2) For risk-neutral users ($r_i = 0$), (20) holds only if
\[ \alpha - c_i^*(\bar{e}_i(b_i^*)) = 0. \] Therefore, $b_i^* = \alpha$, and the platform
induce the users to choose their social optimal efforts. Moreover, since $\alpha$ is the same for all user types, $b_i^*$ would
be the same as well.
3) For risk-seeking users ($r_i < 0$), (20) holds only if
\[ \alpha - c_i^*(\bar{e}_i(b_i^*)) < 0. \] Therefore, $b_i^* > \alpha$, and the platform can
only induce the users to exert efforts higher than $c_i^{opt}$.

**Remark:** Note that we can infer users’ risk attitudes based on
the history of their choices of efforts.

**Complexity Analysis:** Let $\ell$ denote the number of different
types of payment function $f(\cdot)$. For a certain function type, the
time complexity to get its optimal parameters is $O(m)$. Thus
the total time to find the optimal contract overall is $O(\ell m)$.

V. QUAC-I: CONTRACT-BASED INCENTIVE MECHANISM
FOR INCOMPLETE INFORMATION MODEL

In this section, we design an efficient contract-based incentive
mechanism which maximizes the utility of the platform for
the incomplete information model.

**A. Design Rationale**

We formulate the design of contract-based incentive mecha-
nism for the incomplete information model as an optimization
problem with constraints so that both IR and IC properties
are satisfied. To improve the time complexity of computing
the optimal solution, we reduce the number of constraints
and obtain an equivalent optimization problem based on the
inherent properties of the problem.

**B. Design of QUAC-I**

**Optimization Problem Formulation:** We first provide the con-
straints for designing an individual-rational and incentive-
compatible mechanism. Based on the IC property, we must
have $\max_{1 \leq i \leq m} U_{i,j} = U_{i,i}$ for any type-$i$. Then based on (9)
and (10), we use the following constraints, referred to as IR
constraints, to guarantee the IR property,
\[ \theta_i \sum_{k=1}^{m} \pi_{i,k}p_k - q_i \geq \bar{u}, \quad 1 \leq i \leq m. \] (21)
Based on (9) and (11), we use the following constraints, referred to as IC constraints, to guarantee the IC property,
\[ \theta_i \sum_{k=1}^{m} \pi_{i,k}p_k - q_i \geq \theta_j \sum_{k=1}^{m} \pi_{j,k}p_k - q_j, \quad 1 \leq i \neq j \leq m. \] (22)

Note that the actual number of users choosing each contract
item is unknown to the platform a priori. It is impossible to di-
rectly maximize (12) when designing the contract. Therefore,
we use the expected value of $U_P$ as the objective function. In
other words, the objective function is
\[ E[U_P] = \sum_{i=1}^{m} n_i \lambda_i \left( v(q_i) - \sum_{k=1}^{m} \pi_{i,k}p_k \right). \]
Since $n$ is invariant, it can be omitted in the above objective
function. Now we can formulate the mechanism design problem
for the incomplete information model as an optimization
problem below:
\[ \max_{(P,Q)} \sum_{i=1}^{m} \lambda_i \left( v(q_i) - \sum_{k=1}^{m} \pi_{i,k}p_k \right), \quad \text{subject to (21) and (22)}. \]
Note that this optimization problem has $m + m(m-1) = m^2$
constraints in total.

**Constraints Reduction:** Next, we reduce the number of con-
straints and obtain an equivalent problem as follows.

1) **Reducing IR constraints:** By IC constraints and the
relation of use types ($\theta_i > \theta_j$), we have,
\[ \theta_i \sum_{k=1}^{m} \pi_{i,k}p_k - q_i \geq \theta_i \sum_{k=1}^{m} \pi_{1,k}p_k - q_1 \geq \theta_j \sum_{k=1}^{m} \pi_{1,k}p_k - q_1. \]
Therefore, all $m$ IR constraints reduce to only one constraint:
\[ \theta_1 \sum_{k=1}^{m} \pi_{1,k}p_k - q_1 \geq \bar{u}. \] (23)

2) **Reducing IC constraints:** We first introduce the follow-
ing lemma to reduce IC constraints.

**Lemma 1:** For any contract-based incentive mechanism
satisfying both IR and IC properties, we have
\[ \sum_{k=1}^{m} \pi_{i,k}p_k \geq \sum_{k=1}^{m} \pi_{j,k}p_k, \quad \text{for} \quad i > j. \]

**Proof:** By IC constraints, we have
\[ \theta_i \sum_{k=1}^{m} \pi_{i,k}p_k - q_i \geq \theta_j \sum_{k=1}^{m} \pi_{j,k}p_k - q_j \]
and
\[ \theta_j \sum_{k=1}^{m} \pi_{j,k}p_k - q_j \geq \theta_j \sum_{k=1}^{m} \pi_{1,k}p_k - q_1. \]
Adding these two inequalities, we get
\[ \left( \sum_{k=1}^{m} \pi_{i,k}p_k - \sum_{k=1}^{m} \pi_{j,k}p_k \right)(\theta_i - \theta_j) \leq 0. \]
If $i > j$, then $\theta_i > \theta_j$. Thus we have
\[ \sum_{k=1}^{m} \pi_{i,k}p_k - \sum_{k=1}^{m} \pi_{j,k}p_k \geq 0. \]

Among all IC constraints, there are some with certain
special formats. We call the constraints on $U_{i,i}$ and $U_{i,j}$,
where $j < i$, *downward incentive constraints* (DICs). Par-
ticularly, the constraints on $U_{i,i}$ and $U_{i,i-1}$ are called *local
downward incentive constraints* (LDICs). Similarly, we call the constraints on $U_{i,i}$ and $U_{i,j}$, where $j > i$, *upward
incentive constraints* (UICs), and constraints on $U_{i,i}$ and $U_{i,i+1}$ *local
upward incentive constraints* (LUDICs). Note that DICs include
LDICs and that UICs include LUDICs.

Now we show that all LDICs are sufficient to represent all
DICs. By the LDIC on $U_{i,i}$ and $U_{i,i+1}$, we have
\[ \theta_i \sum_{k=1}^{m} \pi_{i,k}p_k - q_i \geq \theta_i \sum_{k=1}^{m} \pi_{i-1,k}p_k - q_{i-1}. \] (24)
By Lemma 1 and $\theta_{i+1} > \theta_i > 0$, we get
\[ \theta_{i+1} \left( \sum_{k=1}^{m} \pi_{i,k}p_k - \sum_{k=1}^{m} \pi_{i-1,k}p_k \right) \]
\[ \geq \theta_i \left( \sum_{k=1}^{m} \pi_{i,k}p_k - \sum_{k=1}^{m} \pi_{i-1,k}p_k \right) \geq q_i - q_{i-1}, \]
where the second inequality is due to (24). Thus,
\[ \theta_{i+1} \sum_{k=1}^{m} \pi_{i,k}p_k - q_i \geq \theta_{i+1} \sum_{k=1}^{m} \pi_{i-1,k}p_k - q_{i-1}. \]
Together with the LDIC on $U_{i+1,i+1}$ and $U_{i+1,i}$, which is based on the original ICs. Thus, we need to make DICs on $\theta$ where the second inequality is due to (25). Thus, we have

$$\theta_{i+1} \sum_{k=1}^{m} \pi_{i+1,k} p_k - q_{i+1} \geq \theta_{i+1} \sum_{k=1}^{m} \pi_{i,k} p_k - q_{i}.$$  

we have

$$\theta_{i+1} \sum_{k=1}^{m} \pi_{i+1,k} p_k - q_{i+1} \geq \theta_{i+1} \sum_{k=1}^{m} \pi_{i-1,k} p_k - q_{i-1}.$$  

Therefore, we have shown that if the LDICs on $U_{i+1,i+1}$ and $U_{i+1,i}$ hold, then the DICs on $U_{i+1,i+1}$ and $U_{i+1,i}$ hold. By extending the above process, we can prove that if all LDICs hold, then all DICs hold.

Next, we show that all LUICs are sufficient to represent all UICs. By the LUIC on $U_{i,i}$ and $U_{i,i+1}$, we have

$$\theta_{i} \sum_{k=1}^{m} \pi_{i,k} p_k - q_{i} \geq \theta_{i} \sum_{k=1}^{m} \pi_{i+1,k} p_k - q_{i+1}.$$  

Moving terms, we get

$$q_{i+1} - q_{i} \geq \theta_{i} \left( \sum_{k=1}^{m} \pi_{i+1,k} p_k - \sum_{k=1}^{m} \pi_{i,k} p_k \right) \geq \theta_{i-1} \left( \sum_{k=1}^{m} \pi_{i+1,k} p_k - \sum_{k=1}^{m} \pi_{i,k} p_k \right),$$

where the second inequality is due to Lemma 1 and the fact that $\theta_{i} > \theta_{i-1}$. By the LUIC on $U_{i-1,i-1}$ and $U_{i-1,i}$, we have

$$\theta_{i-1} \sum_{k=1}^{m} \pi_{i-1,k} p_k - q_{i-1} \geq \theta_{i-1} \sum_{k=1}^{m} \pi_{i,k} p_k - q_{i} \geq \theta_{i-1} \sum_{k=1}^{m} \pi_{i+1,k} p_k - q_{i+1},$$

where the second inequality is due to (25). Thus, we have shown that if the LUICs on $U_{i-1,i-1}$ and $U_{i-1,i}$ hold, then the DICs on $U_{i,i-1}$ and $U_{i+1,i+1}$ hold. By extending the above process, we prove if all LUICs hold, then all DICs hold.

There are $m-1$ LDICs and $m-1$ LUICs in total. In addition, all LDICs can represent all DICs, and all LUICs can represent all UICs. Thus we have successfully reduced $m^2 - m$ IC constraints to $2m - 2$ constraints.

However, during the deduction, we have used Lemma 1 which is based on the original ICs. Thus, we need to make sure the lemma is still valid after the reduction. In fact, by the LDICs $\theta_{i} \sum_{k=1}^{m} \pi_{i,k} p_k - q_{i} \geq \theta_{i} \sum_{k=1}^{m} \pi_{i-1,k} p_k - q_{i-1}$ and the fact $q_{i} \geq q_{i-1}$, we have

$$\theta_{i} \left( \sum_{k=1}^{m} \pi_{i,k} p_k - \sum_{k=1}^{m} \pi_{i-1,k} p_k \right) \geq q_{i} - q_{i-1} \geq 0.$$  

Together with $\theta_{i} > 0$, we get

$$\sum_{k=1}^{m} \pi_{m,k} p_k \geq \sum_{k=1}^{m} \pi_{m-1,k} p_k \geq \cdots \geq \sum_{k=1}^{m} \pi_{1,k} p_k.$$  

Lemma 1 holds as long as LDICs hold.

Simplified Optimization Problem Formulation: Now, the original optimization problem is simplified to the following equivalent convex optimization problem which can be solved through interior-point methods [29]:

$$\max_{(P,Q)} \sum_{i=1}^{m} \lambda_{i} \left( v(q_{i}) - \sum_{k=1}^{m} \pi_{i,k} p_k \right),$$

subject to

$$\theta_{1} \sum_{k=1}^{m} \pi_{1,k} p_k - q_{1} \geq u;$$  

$$\theta_{i} \sum_{k=1}^{m} \pi_{i,k} p_k - q_{i} \geq \theta_{i} \sum_{k=1}^{m} \pi_{i+1,k} p_k - q_{i+1}, 1 \leq i \leq m-1;$$  

$$\theta_{i+1} \sum_{k=1}^{m} \pi_{i+1,k} p_k - q_{i+1} \geq \theta_{i+1} \sum_{k=1}^{m} \pi_{i,k} p_k - q_{i}, 1 \leq i \leq m-1.$$  

Remark: Note that, if the optimal contract results in a negative $U_{p}$, then we will remove users starting from the lowest type since the quality they can provide is the lowest.

Complexity Analysis: The time complexity of QUAC-I is largely based on $v(\cdot)$. For any $v(\cdot)$ that makes the simplified optimization problem become linear programming, quadratic programming, or quadratically constrained quadratic programming, the time complexity can be bound by $O(\sqrt{m})$ [29].

VI. PERFORMANCE EVALUATION

A. Dataset and Evaluation Setup

To evaluate the performance of QUAC-F and QUAC-I, we used the dataset from [30], which is derived from a real dataset [31] to provide the outdoor temperature of areas in Rome collected by 289 taxicabs over 4 days.

In our evaluation, we define the value of quality of sensing data as the submission rate of each taxicab, i.e., how much temperature data submitted to the server per day. Based on the means of qualities, we classify users into 10 different types through $k$-means clustering method [32] and record the variance and the probability of each type. Other characteristics are generated randomly since the raw dataset does not contain those information. The characteristic information of each user type is shown in Table I and the type distribution is shown in Fig. 2. The value shown on the pie chart is the probability that a user belongs to each type. For simplicity, we set the reserved utilities for all users to be $u_t = u = 5$. The valuation function of the platform is set to be $v(q_t) = 6q_t$, which means the platform is willing to pay 6 cents for each temperature value submitted to the server per day. According to [33], 90% tasks on Amazon Mechanical Turk pay less than 10 cents.

For the full information model, we set the cost function of each user to be $c_t(e) = e^2$, which is also used in [34]. Note that here we assume the cost function is the same for any user type for simplicity. We set $n = 1500$ in the evaluation.

For the incomplete information model, the mean $\mu_i$ of the ability of each user type in the full information model is used as the default quality $q_i$. We set the evaluation probability $p_{i,j} = \begin{cases} 0.02, & \text{if } j = i - 1 \text{ or } j = i + 1, \\ 0.96, & \text{if } j = i, \\ 0, & \text{otherwise.} \end{cases}$

We vary the number of users $n$ from 300 to 1500 in this model.

The performance metrics used in our evaluation are: data quality, user effort, user utility, and platform utility.

B. Evaluation of QUAC-F

Table II shows the optimal contract designed by QUAC-F.
Fig. 2: User type distribution

<table>
<thead>
<tr>
<th>User type</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-1</td>
<td>11%</td>
</tr>
<tr>
<td>Type-2</td>
<td>13%</td>
</tr>
<tr>
<td>Type-3</td>
<td>9%</td>
</tr>
<tr>
<td>Type-4</td>
<td>11%</td>
</tr>
<tr>
<td>Type-5</td>
<td>8%</td>
</tr>
<tr>
<td>Type-6</td>
<td>12%</td>
</tr>
<tr>
<td>Type-7</td>
<td>13%</td>
</tr>
<tr>
<td>Type-8</td>
<td>10%</td>
</tr>
<tr>
<td>Type-9</td>
<td>9%</td>
</tr>
<tr>
<td>Type-10</td>
<td>4%</td>
</tr>
</tbody>
</table>

Table I: Characteristics of each user type

<table>
<thead>
<tr>
<th>Type</th>
<th>µ_i</th>
<th>σ_{µ_i}</th>
<th>σ_{r_i}</th>
<th>r_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-1</td>
<td>1.08</td>
<td>0.19</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td>Type-2</td>
<td>2.08</td>
<td>0.03</td>
<td>0.06</td>
<td>-0.35</td>
</tr>
<tr>
<td>Type-3</td>
<td>2.60</td>
<td>0.02</td>
<td>0.04</td>
<td>0.84</td>
</tr>
<tr>
<td>Type-4</td>
<td>3.21</td>
<td>0.04</td>
<td>0.09</td>
<td>-0.10</td>
</tr>
<tr>
<td>Type-5</td>
<td>3.99</td>
<td>0.04</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Type-6</td>
<td>4.77</td>
<td>0.03</td>
<td>0.10</td>
<td>0.04</td>
</tr>
<tr>
<td>Type-7</td>
<td>5.58</td>
<td>0.08</td>
<td>0.05</td>
<td>0.32</td>
</tr>
<tr>
<td>Type-8</td>
<td>6.62</td>
<td>0.09</td>
<td>0.08</td>
<td>0.27</td>
</tr>
<tr>
<td>Type-9</td>
<td>7.82</td>
<td>0.15</td>
<td>0.08</td>
<td>0.18</td>
</tr>
<tr>
<td>Type-10</td>
<td>9.64</td>
<td>1.61</td>
<td>0.09</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Fig. 3: User effort of each user type under different risk attitudes

Fig. 4: Data quality for each user type with/without the incentive mechanism

Table II: Contract designed by QUAC-F

<table>
<thead>
<tr>
<th>User type</th>
<th>q_i</th>
<th>a_i</th>
<th>b_i</th>
<th>C_{µ_i}</th>
<th>C_{r_i}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-1</td>
<td>3.94</td>
<td>5.49</td>
<td>5.11</td>
<td>6.38</td>
<td>6.98</td>
</tr>
<tr>
<td>Type-2</td>
<td>13.08</td>
<td>-26.01</td>
<td>-27.70</td>
<td>-31.49</td>
<td>-29.68</td>
</tr>
<tr>
<td>Type-3</td>
<td>5.72</td>
<td>6.81</td>
<td>5.01</td>
<td>6.35</td>
<td>5.97</td>
</tr>
<tr>
<td>Type-4</td>
<td>-8.98</td>
<td>-22.17</td>
<td>-13.08</td>
<td>-12.17</td>
<td>-10.35</td>
</tr>
<tr>
<td>Type-5</td>
<td>5.01</td>
<td>6.35</td>
<td>5.97</td>
<td>5.48</td>
<td>5.19</td>
</tr>
<tr>
<td>Type-6</td>
<td>10.35</td>
<td>12.72</td>
<td>10.35</td>
<td>9.64</td>
<td>9.23</td>
</tr>
<tr>
<td>Type-7</td>
<td>11.08</td>
<td>13.08</td>
<td>11.08</td>
<td>9.64</td>
<td>9.23</td>
</tr>
<tr>
<td>Type-8</td>
<td>12.72</td>
<td>15.12</td>
<td>12.72</td>
<td>12.72</td>
<td>12.72</td>
</tr>
<tr>
<td>Type-9</td>
<td>15.12</td>
<td>17.52</td>
<td>15.12</td>
<td>17.52</td>
<td>17.52</td>
</tr>
<tr>
<td>Type-10</td>
<td>22.07</td>
<td>25.72</td>
<td>22.07</td>
<td>25.72</td>
<td>25.72</td>
</tr>
</tbody>
</table>

Table III: Contract designed by QUAC-I

<table>
<thead>
<tr>
<th>User type</th>
<th>q_i</th>
<th>a_i</th>
<th>b_i</th>
<th>C_{µ_i}</th>
<th>C_{r_i}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-4</td>
<td>1.08</td>
<td>2.08</td>
<td>2.60</td>
<td>3.21</td>
<td>3.99</td>
</tr>
<tr>
<td>Type-7</td>
<td>14.51</td>
<td>16.49</td>
<td>17.03</td>
<td>17.94</td>
<td>19.00</td>
</tr>
<tr>
<td>Type-10</td>
<td>19.98</td>
<td>20.92</td>
<td>21.90</td>
<td>23.31</td>
<td>25.72</td>
</tr>
</tbody>
</table>

Fig. 5: Platform utility of each individual user type

Fig. 6: Utility of user when choosing different contract items

Fig. 7: Utility of each user type under different contracts

Fig. 8: Utility of the platform under different contracts

Table III shows the optimal contract designed by QUAC-I. Table III shows the optimal contract designed by QUAC-I. Table III shows the optimal contract designed by QUAC-I.

C. Evaluation of QUAC-I

Fig. 8 plots the utility of the platform under different contracts designed by QUAC-I. As expected, QUAC-I achieves higher utility when a user has a higher ability except for type-2 users. This is because these users are incentivized to exert higher effort than type-2 users as shown in Fig. 4.
the platform will set very low payments so as to maximize its utility. Therefore, the users would have negative utilities since the payments are lower than their costs. Without IC property, \( U_{i,1} \) under NO-IC is always the reserved utility no matter what type the user is. We also observe that \( U_{i,1} = u \) under QUAC-I. The reason is that the IR constraint for type-1 users must bind in the optimal solution.

Fig. 8 shows the impact of \( n \) on the platform utility, \( U_P \), under the above three mechanisms. Without IR, users would have utility less than \( u \) if they accept contracts, and thus they would reject all contracts. Therefore, the platform utility is always zero. Without IC property, the platform utility under NO-IC is lower than that under QUAC-I since the all users will tend to choose the contract item 1 since they can achieve the same utilities with the lowest quality. However, such quality has the least contribution to the platform.

Remark: We have shown how QUAC-F and QUAC-I incentivize taxi drivers to increase their submission rates. Our mechanisms can also be applied to any crowdsensing systems as long as the quality of the sensing tasks can be quantified and the users’ mathematical model can be established. Such quantification and formulation is largely dependent of the crowdsensing system and is orthogonal to our research. Fortunately, existing works [35] may help to shed light upon those researches.

VII. CONCLUSION

In this paper, we studied the design of quality-aware contract-based incentive mechanisms for crowdsensing. Based on different levels of the platform’s knowledge on users’ information, we introduced two models, the full information model and the incomplete information model. In the full information model, the platform knows the basic characteristics (e.g. ability distribution, cost function, risk attitude, etc) of users but does not know their action (how much effort to contribute). Moreover, we considered the uncertainty in the intended qualities of users’ submitted sensing data with their efforts. In the incomplete information model, however, the platform only knows the distribution of users’ general characteristics (type value and default quality). We also considered imperfect quality evaluation system in this model. For both models, we design quality-aware contract-based incentive mechanisms named QUAC-F and QUAC-I respectively, which are guaranteed to be individual-rational and incentive-compatible while maximizing the platform utility. We also plan to test the effectiveness of our mechanisms by implementing them on crowdsensing platforms such as Amazon Mechanical Turk.

REFERENCES