Robust Incentive Tree Design for Mobile Crowdsensing

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Abstract—With the proliferation of smart mobile devices (smart phone, tablet, and wearable), mobile crowdsensing becomes a powerful sensing and computation paradigm. It has been put into application in many fields, such as spectrum sensing, environmental monitoring, healthcare, and so on. Driven by promising incentives, the power of the crowd grants crowdsensing an advantage in mobilizing users who perform sensing tasks with the embedded sensors on the smart devices. Auction is one of the commonly adopted crowdsensing incentive mechanisms to incentivize users for participation. However, it does not consider the incentive for user solicitation, where in crowdsensing, such incentive would ease the tension when there is a lack of crowdsensing users. To deal with this issue, we aim to design an auction-based incentive tree to offer rewards to users for both participation and solicitation. Meanwhile, we want the incentive mechanism to be robust against dishonest behavior such as untruthful bidding and sybil attacks, to eliminate malicious price manipulations. We design RIT as a Robust Incentive Tree mechanism for mobile crowdsensing which combines the advantages of auctions and incentive trees. We prove that RIT is truthful and sybil-proof with probability at least $H$, for any given $H \in (0, 1)$. We also prove that RIT satisfies individual rationality, computational efficiency, and solicitation incentive. Simulation results of RIT further confirm our analysis.

Key Words: Crowdsensing; Mobile Networks; Wireless Networks; Incentive Mechanism; Sybil-proofness; Truthfulness.

1. INTRODUCTION

With the proliferation of smart mobile devices (e.g., smart phone, tablet, and wearable), the richness of embedded sensors (e.g., accelerometer, compass, gyroscope, GPS), and the increasingly powerful processors, mobile users can perform basic sensing tasks single-handedly. Crowdsensing [10], which falls into the category of mobile crowdsourcing [15], has blossomed into a fertile area for commercial business, where mobile device users are recruited to collectively share their sensing data. Unlike traditional labor markets where jobs are assigned to designated workers, crowdsensing out-sources the jobs to the crowd who can be recruited online or through social networks. Driven by promising incentives, the power of the crowd grants crowdsensing advantages in mobilizing crowdsensing users to provide services in various areas, such as spectrum sensing [1], environmental monitoring [19], traffic prediction [42], healthcare [14, 28, 30], and so on.

Behind the success of these applications, one of the forces that motivate the users into participation is the incentives paid to the crowdsensing users. When performing crowdsensing jobs, it incurs costs to crowdsensing users, in terms of power consumption, private information leakage, time spent, etc. These costs might prevent the users from further participation. Thus, monetary rewards would be offered to the crowdsensing users as compensations such that despite the costs, users would still devote their efforts to perform the crowdsensing jobs.

A good incentive mechanism, which allocates tasks and computes proper payments, is crucial to the success of a crowdsensing application. Auction is one of the commonly adopted incentive mechanisms for crowdsensing [9, 17, 20, 23, 35, 40]. There are several desired properties for an auction: truthfulness, individual rationality, and computational efficiency. These properties aim to make the mechanism robust, fair, and executable. We will present formal definitions of these properties in Section 3-C. Among these properties, truthfulness is the crowning jewel which guarantees the robustness of the auction, such that each user is better off being honest to reveal its true cost. If truthfulness is not guaranteed for an auction, dishonest users may misreport their sensing costs and receive payments higher than they deserve, and honest users may leave the crowdsensing platform in fear of price manipulation.

Though being widely-adopted, auction has its limitation. In a crowdsensing auction, it is assumed that all users are known to be present and are aware of the existence of the auction. However, this is not always true for crowdsensing. In many scenarios, finishing a crowdsensing job requires a large amount of users’ efforts, and the efforts from existing users are far from enough. Thus, auction alone cannot satisfy the need when there are not enough users to perform the sensing tasks. One possible solution is to provide extra incentives to users who could recruit more users for the crowdsensing job.

On the other hand, incentive tree mechanisms are designed [3, 6–8, 24] to incentivize individuals for solicitation. An incentive tree is a tree-structured incentive mechanism which offers rewards to each individual for solicitation. Incentive tree mechanisms can be applied to provide incentives for existing users to spread the word to other potential users hidden in their social networks. It can depict the solicitation process of the crowdsensing users (if user $P_1$ refers user $P_2$, then in the incentive tree $P_1$ is the parent of $P_2$), and computes the payment for each user based on its own contribution and the contribution from its descendants.
A perfect poster child of the incentive tree mechanisms is the Pyramid Scheme [27], which provides promising rewards for solicitation. The DARPA Network Challenge [4] is another example where the incentive tree demonstrated its efficiency in crowdsourcing. In 2009, DARPA initiated the challenge where contestants were asked to locate ten balloons deployed across the United States with a $40,000 reward for the winner. An MIT team [26] won the challenge by applying an incentive tree mechanism which recruited nearly 4,400 participants within nine hours to find all ten balloons. Their strategy is as follows. Each balloon finder is rewarded with $2,000. The inviter of the finder is rewarded with $1,000, and the one who refers the inviter is rewarded with $500, and so on. This mechanism not only mobilizes participants to locate the balloons, but to further refer others into action. Using similar strategies, the incentive tree has shown its efficiency in many contests such as the TAG Challenge [29] and the DARPA Shredder Challenge [5].

There is a drawback in these incentive tree mechanisms despite their efficiencies—they are not robust against sybil attacks. A sybil attack is that a user may generate multiple fake identities, and gain a higher utility without devoting extra efforts. E.g., in the MIT strategy of the DARPA Network Challenge, Bob is the balloon finder and Alice is the inviter of Bob. Then Bob receives $2,000 and Alice receives $1,000. Now suppose that Bob launches a sybil attack and splits himself into Bob1 and Bob2, where Bob1 is the balloon finder, Bob2 is the inviter of Bob1, and Alice is the inviter of Bob2. According to the payment rule, Bob1 receives $2,000, Bob2 receives $1,000, and Alice receives $500. Then Bob will receive $2,000 + $1,000 = $3,000 comparing to what he deserves ($2,000). On the other hand, Alice will receive $500 comparing to what she deserves ($1,000). From this example, we know that without robustness against sybil attacks, a dishonest user may increase its utility with no extra efforts devoted, and an honest user may not receive the utility that she deserves. Thus, an incentive tree mechanism should not only offer rewards for solicitation, but be robust against sybil attacks (and this property is named as sybil-proofness).

To design an incentive mechanism that incentivizes both participation and solicitation with truthfulness and sybil-proofness, we cannot simply combine an existing sybil-proof incentive tree with an existing truthful auction, as these two properties may bring new issues to each other. For instance, there may be coalitions from the identities generated by one dishonest user. This coalition may violate the truthfulness of the mechanism as most crowdsensing auctions are not designed for robustness against coalitions. Furthermore, it is proved that no deterministic auction can be robust against coalitions [12]. Thus, we turn to randomized algorithms for a satisfactory probability of truthfulness and sybil-proofness. Therefore, it is non-trivial to design a crowdsensing incentive mechanism which is truthful and meanwhile sybil-proof.

We propose RIT (Robust Incentive Tree Mechanism for Crowdsensing) to incentivize users for both participation and solicitation. We prove that RIT is truthful and sybil-proof with probability at least \( H \), for any given \( H \in (0, 1) \). We also prove that RIT satisfies individual rationality, computational efficiency, and solicitation incentive. We implemented RIT and the extensive evaluations further confirm our analysis.

The main contributions of this paper are:

- To the best of our knowledge, we initiate the problem of designing crowdsensing incentive mechanisms which incentivize users for both participation and solicitation, while guaranteeing truthfulness and sybil-proofness.
- We design RIT as the first robust auction-based incentive tree mechanism for crowdsensing. We prove that RIT is truthful and sybil-proof with probability at least \( H \), for any given \( H \in (0, 1) \). We also prove that RIT satisfies individual rationality, computational efficiency, and solicitation incentive.

The remainder of this paper is organized as follows. In Section 2, we review the state-of-art works on truthful crowdsensing auctions and sybil-proof incentive tree mechanisms. In Section 3, we describe the crowdsensing model, formulate the problem studied in this paper, and present the design goal. In Section 4, we illustrate the challenges when designing truthful and sybil-proof incentive mechanisms. We present RIT and its analysis in Section 5 and Section 6, respectively. In Section 7, we present and analyze the extensive performance evaluation results. We draw our conclusions in Section 8.

2. RELATED WORK

There is an abundance of research efforts on truthful auction design for crowdsensing. Two mobile crowdsensing models were proposed by Yang et al. [35], where a unique Stackelberg Equilibrium was computed in the first and a truthful auction was designed for the second. Jin et al. [17] proposed a truthful and quality-aware crowdsensing auction which approximates the social welfare. Wen et al. [34] designed a quality-driven auction for crowdsensing which guarantees truthfulness and maximizes the social welfare. A truthful auction for crowdsensing was proposed by Koutsopoulos [20] to determine the participation levels and the payments. Luo et al. [23] designed an all-pay auction which approximates the utility of the organizer in crowdsensing. Feng et al. studied auctions for location-aware crowdsensing [9]. Wang et al. [32] designed a quality-aware truthful auction for crowdsensing, which minimizes the expected expenditure. There are also existing works on collusion-resistant auctions. Goldberg and Hartline [12] studied truthful auctions using a consensus rounding technique to achieve truthfulness against coalitions with a high probability. Following this line, Wang et al. [33] applied multiple rounds of an algorithm \( AEM \) from [12] to allocate resources in cloud computing. Zhou and Zheng [43] combined collusion-resistance with spatial reusability for cognitive radio networks. Zhang et al. [38] designed a truthful and sybil-proof auction for crowdsourcing. However, none of these auctions considers providing incentives for solicitation.
As for the sybil-proof incentive trees, Douceur and Moscibroda [6] proposed Pachira Lottree for distributed service installations, which is sybil-proof. Emek et al. [8] proposed a sybil-proof diffusion mechanism within a social network. Two families of sybil-proof incentive trees were proposed by Lv and Moscibroda [24], where each satisfies a maximal subset of the desired properties. Following this line, Zhang et al. [37] proposed a time-sensitive and sybil-proof incentive tree for crowdsourcing. Drucker and Fleischer [7] proposed a family of sybil-proof mechanisms by modifying incentive mechanisms with natural properties. Chen et al. [3] proposed a direct referral tree under the query incentive networks. None of these incentive tree mechanisms considers using truthfulness to guide the behavior of each participant.

3. Crowdsensing Model and Problem Formulation

In this section, we present the crowdsensing model, describe the dishonest behaviors, and state the design goal.

A. Crowdsensing Model

In the crowdsensing model, there is a crowdsensing platform who has a sensing job \( J \) to be finished. \( J \) consists of many sensing tasks that can be completed by a crowdsensing user single-handedly. Each sensing task is indivisible. We categorize the tasks into \( m \) types: \( \tau_1, \tau_2, \ldots, \tau_m \), according to their locations. For instance, in mobile spectrum sensing, users are required to sense the spectrum usage in two different areas, where each area contains several points of interest (POIs) to be sensed by the users. We regard the sensing in each area as one task type, and regard the sensing at one POI as one task. We use \( \Gamma \) to denote the set \( \{\tau_1, \tau_2, ..., \tau_m\} \).

There is a set of \( n \) crowdsensing users \( P = \{P_1, P_2, ..., P_n\} \) who can provide services to complete tasks. Each crowdsensing user \( P_j \) chooses one area (task type) \( t_j \in \Gamma \) to perform the sensing tasks. In mobile spectrum sensing, this relates to the geographical location of the user, where each user has difficulties to sense the spectrum usage in two different areas in the same time window. Each user \( P_j \) can complete at most \( K_j \geq 0 \) tasks in type \( t_j \in \Gamma \). E.g., if \( P_1 \) can complete at most three tasks in type \( \tau_2 \), then \( t_1 = \tau_2 \) and \( K_1 = 3 \). Meanwhile, \( P_j \) has a private unit cost \( c_j > 0 \) to complete one task in type \( t_j \). We define the largest \( K_j \) as \( K_{\text{max}} \), i.e., \( K_{\text{max}} = \max_{P_j \in P} \{K_j\} \).

The job \( J \) is described as a multi-subset of \( \Gamma \) and it is finished if and only if all tasks in \( J \) are completed. We use \( m_i \) to denote the number of tasks in type \( \tau_i \) requested by \( J \). For instance, if there is a crowdsensing job \( J = \{\tau_1, \tau_2, \tau_3, \tau_3\} \), we have \( m = 3, m_1 = 1, m_2 = 1, \) and \( m_3 = 2 \). \( J \) is finished if and only if one task in type \( \tau_1 \), one task in type \( \tau_2 \), and two tasks in type \( \tau_3 \) are completed.

The crowdsensing model works as follows. The platform posts the sensing job \( J \). After \( J \) is posted, several users would join and perform the sensing tasks. However, these users may not be able to finish the job (perform all sensing tasks) by themselves. Thus, the platform offers extra incentives such that the existing users would further refer other users to join them. We use an incentive tree \( T \) to depict this solicitation process, where each user is represented by a node in the tree. There is an edge from \( P_i \) to \( P_j \) if \( P_j \) joins by the solicitation of \( P_i \). To make the structure of \( T \) a tree instead of a forest, we set the crowdsensing platform as the root and the users who join at the very beginning are the children of the root. When the number of crowdsensing users reaches a threshold value \( N \), \( T \) stops growing and the solicitation comes to an end. We will discuss how to choose the threshold value of \( N \) in Remark 6.1. Upon joining the incentive tree, each user will notify the platform from whom it is solicited. Thus, the structure of \( T \) is known to the platform at the end of solicitation. We use \( T_j \) to denote the set of nodes who are descendants of \( P_j \), and \( r_j \) to denote the distance from \( P_j \) to the root.

Upon joining the incentive tree, each user \( P_j \) submits an ask \( (t_j, k_j, a_j) \) to the platform, where \( k_j > 0 \) is the maximum number of tasks that \( P_j \) claims to be able to complete, and ask value \( a_j > 0 \) is the minimum amount of reward that \( P_j \) requires to complete one task in type \( t_j \). This submission is seal-bid, which means that by the time of submission, no user is aware of the ask from any other user. Note that \( k_j \) is not necessarily equal to \( K_j \), but we assume that \( k_j \leq K_j \) since \( P_j \) is not able to complete more than \( K_j \) tasks. Furthermore, \( a_j \) is not necessarily equal to \( c_j \), since \( P_j \) may gain a higher utility by not revealing its cost. We use \( A \) to denote the ask vector \( \{(t_1, k_1, a_1); (t_2, k_2, a_2); ...; (t_N, k_N, a_N)\} \).

After collecting the asks, the crowdsensing platform first computes an auction payment \( p_j^A \) for each user \( P_j \). The platform also computes the indicator \( x_j \) for \( P_j \) which indicates the number of tasks in type \( t_j \) that is allocated to \( P_j \). Combining the auction payments and the incentive tree \( T \), the platform computes the final payment \( p_j \) for each user \( P_j \), which is the actual amount that the platform pays to \( P_j \). Note that the auction payment is not the payment that each user receives. We only use it to compute the final payment \( p_j \). We use \( p^A, p, \) and \( x \) to denote vectors \( (p_1^A, p_2^A, ..., p_N^A), (p_1, p_2, ..., p_N) \), and \( (x_1, x_2, ..., x_N) \), respectively.

The utility of user \( P_j \) with ask \( (t_j, k_j, a_j) \) is defined as \( U_j(t_j, k_j, a_j) = p_j - x_j c_j \), which is \( P_j \)'s payment minus its incurred cost.

B. Dishonest Behaviors

We consider two situations that a user \( P_j \) may gain a higher utility from being dishonest. The first one is that \( P_j \) may submit an ask value \( a_j \) deviating from its cost \( c_j \). The second one is that \( P_j \) may generate fake identities to manipulate the auction payments or to increase the rewards from the incentive tree. We use truthfulness to incentivize \( P_j \) to reveal its cost \( c_j \). To prevent \( P_j \) from generating fake identities, we first give a formal description of how a sybil attack is launched before
introducing the concept of sybil-proofness.

A sybil attack is that a user \( P_j \) may generate \( \delta(j) \) fake identities: \( P_{j1}, P_{j2}, \ldots, P_{j\delta(j)} \), where \( \delta(j) > 1 \). An identity of \( P_j \) resides in the incentive tree either as a child of \( P_j \)'s parent or as a child of another identity of \( P_j \). It does not attach to other users because the other users didn’t reach out for \( P_j \) during the solicitation. For each child of \( P_j \) in the original incentive tree \( T \), it is attached to one of \( P_j \)'s identities after the sybil attack, while the other parts of the incentive tree remain unchanged.

**Remark 3.1:** When defining sybil attacks, we attach an identity of \( P_j \) to \( P_j \)'s parent or \( P_j \)'s other identities as a technical convention of sybil attacks [6–8, 24].

Each identity of \( P_j \), denoted as \( P_{j_i} \), acts as a user who submits an ask \( (t_{j_i}, k_{j_i}, a_{j_i}) \), where \( t_{j_i} \), \( k_{j_i} \), and \( a_{j_i} \) have similar definitions to those of \( t_j \), \( k_j \), and \( a_j \), respectively. We assume that \( P_j \)'s identities’ asks do not exceed \( P_j \)'s capability, i.e., \( t_{j_i} = t_j \) and \( \sum_{l=1}^{\delta(l)} k_{l_i} \leq K_j \). Thus, the ask \( (t_{j_i}, k_{j_i}, a_{j_i}) \) can be written as \( (t_j, k_i, a_i) \). The unit cost of \( P_{j_i} \) is \( c_{j_i} = c_j \).

![Fig. 1. A sybil attack from \( P_2 \)](image)

Fig. 1 presents an example of a sybil attack from \( P_2 \) with ask \((\tau_2, 5, 7)\). \( P_2 \) generates three identities \( P_{21}, P_{22}, \) and \( P_{23} \) with asks \((\tau_2, 1, 4), (\tau_2, 2, 6), \) and \((\tau_2, 2, 8), \) respectively.

Similar to the definition of \( P_j \)'s utility in Equation, \( P_j \)'s utility is defined as \( U_{j_i}(t_j, k_j, a_j) = p_{j_i} - x_j c_j \), where \( p_{j_i} \) and \( x_j \) have similar definitions to those of \( p_j \) and \( x_j \), respectively. The utility of \( P_{j_i} \) from a sybil attack is \( \sum_{l=1}^{\delta(j)} p_{l_i} - \sum_{l=1}^{\delta(j)} x_j c_j = \sum_{l=1}^{\delta(j)} U_{j_i}(t_j, k_j, a_j) \).

We claim that for each \( P_j \), it could not generate more than \( K_{\text{max}} \) fake identities. This is because for each fake identity \( P_j, K_j \geq k_{j_i} \geq 1 \). Thus, there will be no more than \( K_j \) fake identities of \( P_j \). This claim is closely related to the design goal introduced in Section 3-C.

**C. Desired Properties and Design Goal**

The concepts of the desired properties are presented as follows.

- **Truthfulness:** No user could increase its utility by reporting an ask value other than its cost, i.e., \( U_j(t_j, k_j, c_j) \geq U_j(t_j, k_j, a_j) \);

- **Sybil-Proofness:** No user could benefit from generating multiple fake identities, i.e., \( U_j(t_j, k_j, c_j) \geq \sum_{l=1}^{\delta(j)} U_{j_i}(t_j, k_j, a_j) \), where \( k_j \geq \sum_{l=1}^{\delta(j)} k_{l_i} \), for any \( k_{j_i} \) and \( \delta(j) \);

- **Individual Rationality:** No user has a negative utility by revealing its cost, i.e., \( U_j(t_j, k_j, c_j) \geq 0 \);

- **Computational Efficiency:** The mechanism can be executed within polynomial time;

- **Solicitation Incentive:** If \( P_j \) is about to join \( T \), then \( P_j \)'s utility when \( P_j \) joins is no less than \( P_j \)'s utility when \( P_j \) joins as another user’s child.

There are many other properties that receive interests from research communities when designing crowdsensing incentive mechanisms, such as data quality guarantee [11, 36] and privacy protection [16, 18, 22]. These properties are out of the scope of this paper and are subject to future research.

In this paper, we want to design an incentive mechanism to guarantee both truthfulness and sybil-proofness, i.e., \( U_j(t_j, K_j, c_j) \geq \sum_{l=1}^{\delta(j)} U_{j_i}(t_j, k_j, a_j) \). However, we cannot simply combine a sybil-proof incentive tree with an existing truthful crowdsensing auction to achieve this. The reasons are two-folded. On one hand, by lying about the asks, an identity of a user may increase the payment of another identity in the incentive tree. On the other hand, multiple identities from the same user may form coalitions to violate truthfulness of the mechanism, since most truthful crowdsensing auctions do not consider such coalitions.

To overcome such difficulty, we take the inspiration from a consensus rounding technique [12], and develop new schemes to guarantee truthfulness against coalitions with a high probability. By defining high probability, it regards to some parameter of the input, e.g., the number of asks and the number winners in the auction. We introduce the following concepts.

- **\( d \)-truthfulness:** For any coalition of size at most \( d \), the total utility of the coalition could not be increased if some of them reveal their ask values other than their costs;

- **\( (d, \eta) \)-truthfulness:** A mechanism is \( (d, \eta) \)-truthful if it is \( d \)-truthful with probability at least \( \eta \).

Since \( P_j \) could not generate more than \( K_{\text{max}} \) fake identities, the incentive mechanism needs to be \( K_{\text{max}} \)-truthful for each task type. It has been proved in [12] that no deterministic auction can achieve \( K_{\text{max}} \)-truthfulness when \( K_{\text{max}} \geq 2 \). Thus, our goal is to design a \((K_{\text{max}}, H)\)-truthful mechanism.

**The design goal of this paper:** under the crowdsensing model presented in Section 3-A, given a probability \( H \in (0, 1) \), design an incentive mechanism such that for each crowdsensing user \( P_j \), \( U_j(t_j, K_j, c_j) \geq \sum_{l=1}^{\delta(j)} U_{j_i}(t_j, k_j, a_j) \) for any \( K_j \) and \( \delta(j) \) with probability at least \( H \). This encourages \( P_j \) to reveal \( K_j \) and \( c_j \), and not launch sybil attacks. The mechanism should meanwhile guarantee individual rationality, computational efficiency, and solicitation incentive.

**4. Design Challenges**

In Section 3-C, we have briefly introduced the difficulty to achieve both truthfulness and sybil-proofness. In this section, we illustrate the challenges in detail when designing the
incentive mechanism. We focus on the impact of truthfulness and sybil-proofness on each other. Through these discussions, we show that we cannot simply combine an existing truthful auction and an existing sybil-proof incentive tree mechanism. Thus, it is non-trivial to design a mechanism that achieves both truthfulness and sybil-proofness.

A. Impact of Auctions on Sybil-proofness

Most sybil-proof incentive mechanisms are contribution-based [2, 6, 7, 24], where payments are computed based on the contribution from the users. In this paper, we use the auction payment to quantify the contribution of each user, since payment itself is a measurement of the contribution.

For existing sybil-proof mechanisms, they are robust against multi-identity attacks, such that if a user launches an attack, it will not have an increment in utility. However, if we combine the auctions with a sybil-proof incentive tree mechanism, the sybil-proofness might be violated.

![Fig. 2. An illustration of the impact from the auctions on sybil-proofness](image)

We use the Fig. 2 to provide an example, where there are three users $P_1, P_2$, and $P_3$ with truthful asks $(\tau_1, 2, 2), (\tau_1, 1, 3)$, and $(\tau_1, 1, 5)$, respectively. The job $J$ requires two tasks of type $\tau_1$.

For ease of understanding, we use the $k$-th lowest price auction instead of the complicated truthful crowdsensing auctions for this example. In the $k$-th lowest price auction, there are several bidders, each of whom sells an item (or service). Each bidder has a private cost and submits an ask. The winners are the ones who submit the $k - 1$ lowest asks, and their payments are the $k$-th lowest ask. Each bidder’s utility is the payment received minus its cost. It has been proven that the $k$-th lowest price auction is a truthful auction [31]. The sybil-proof incentive tree mechanism that we use to compute the final payment is from [24], where the reward for $P_j$ is $p_j = 2p_j^A + \ln (1 - \frac{p_j^A}{\sum_{P_i \in \tau_j} p_i^A})$. (We use the auction payment as the contribution of each user.)

We focus on $P_1$. If each user submits its truthful ask, $P_1$ is assigned to complete two tasks, and the auction payment is $2 \times 3 = 6$. Thus, $p_1 = 5.85$ and $P_1$’s utility is $5.85 - 2 \times 2 = 1.85$. Suppose that $P_j$ launches a multi-identity attack as shown in Fig. 2. Then $P_1$ is assigned with one task and its auction payment is $5$. $P_2$ is assigned with the other task and its auction payment is $5$. Thus, $p_{12} = 4.39, p_{13} = 0$, and $P_1$’s utility is $4.39 - 2 = 2.39 > 1.85$. From this example, we know that combining auctions with sybil-proof incentive trees may violate sybil-proofness.

B. Impact of Incentive Trees on Truthfulness

There is a large body of efforts on truthful incentive mechanisms for crowdsensing [9, 17, 20, 23, 35, 36, 39–41]. However, if we combine these mechanisms with incentive trees, where users have incentives for solicitation, the truthfulness of these mechanisms may be violated since the payment is also related to the solicitation incentive.

We use the instance in Fig. 3 to illustrate such impact.

![Fig. 3. An illustration of the impact from the incentive trees on truthfulness](image)

In Fig. 3, there are four bidders $P_1, P_2, P_3$, and $P_4$ who sell services for one task type $\tau_1$, with cost values 5, 4, 5, and 4, respectively. The job $J$ requires two tasks of type $\tau_1$. We use the third price auction and the sybil-proof incentive tree in [24] to compute the final payment, where the reward for $P_j$ is $p_j = 2p_j^A + \ln (1 - \frac{p_j^A}{\sum_{P_i \in \tau_j} p_i^A})$.

We focus on user $P_1$. If each user submits its truthful ask, $p_1^A = 0, p_1 = 0$, and $P_1$’s utility is 0. If $P_1$ bids $4 - \epsilon$ instead, then $p_1^A = 4, p_1 = 7.41$, and $P_1$’s utility is $7.41 - 5 = 2.41 > 0$. Thus, $P_1$ has the incentive to bid untruthfully, which indicates that incentive trees may violate the truthfulness of an auction. Furthermore, when applying incentive trees on truthful auctions, the auction payment may also be influenced if there is a multi-identity attack. This is because the auction payment of one identity depends on the asks from other identities of the same user.

5. DESIGN OF RIT

In this section, we present the incentive mechanism RIT (Robust Incentive Tree Mechanism for Crowdsensing).

A. Design Rationale

The mechanism RIT consists of two phases: the auction phase and the payment determination phase. The auction phase allocates tasks to each user $j$ and computes the auction payment $p_j^A$. In the auction phase, we propose two algorithms: CRA (Collusion Resistant Auction) in Algorithm 1 as a basic auction, which allocates at most $m_i$ tasks to users and guarantees $k$-truthfulness with a high probability, and Extract in Algorithm 2, which converts the asks from the users into the format that CRA requires. To make sure that exactly $m_i$ tasks are assigned to the users for each $\tau_i$, we run multiple
rounds of CRAs to allocate the tasks and meanwhile guarantee $(K_{\text{max}}, H)$-truthfulness. In the payment determination phase, based on the auction payment $p_j^s$ and the solicitation of user $j$ in the incentive tree, we propose an incentive tree mechanism to compute $p_j$.

B. Design Details

We first present CRA in Algorithm 1 and Extract in Algorithm 2 which are used by RIT in Algorithm 3.

We design CRA in Algorithm 1 to allocate at most $q$ tasks in type $\tau_i$. Let $\alpha = (\alpha_1, \alpha_2, \ldots)$ be a vector of asks where each $\alpha_\omega$ bids for one task in type $\tau_i$. Let $x'_\omega$ be the indicator such that if $\alpha_\omega$ is allocated to one task in CRA, $x'_\omega = 1$; otherwise, $x'_\omega = 0$. Let $p'_\omega$ be the auction payment for $\alpha_\omega$. CRA takes the ask vector $\alpha$ for type $\tau_i$, the number of unallocated tasks (in $\tau_i$) $q$, and $m_\omega$ as input, and outputs the indicator vector $x' = (x'_1, x'_2, \ldots)$ and payment vector $p' = (p'_1, p'_2, \ldots)$. From Line 1 to Line 16 of Algorithm 1, we select at most $q + m_\omega$ asks as potential winning asks. If there are more than $q$ asks selected, we randomly choose $q$ asks among them with equal probability as the winning asks and allocate one task to each winning ask. The reason why we first select no more than $q + m_\omega$ potential winning asks and then choose $q$ winning asks is to guarantee that the auction phase is $K_{\text{max}}$-truthful with high probability. We will provide more detailed explanations in Lemma 6.2 and Remark 6.1.

In Algorithm 1, we first sample a subset $S$ of asks from $\alpha$ randomly with equal probability $\frac{1}{q + m_\omega}$ in Line 2. We define $s$ as the smallest sampled ask value in Line 3. Next we calculate $n_s$ based on its definition in Line 5. If $n_s \leq q + m_\omega$, we temporarily choose the smallest $n_s$ asks in $\alpha$ in Line 7. Otherwise, among the smallest $n_s$ asks, we choose each one with equal probability $\frac{q + m_\omega}{2^{m_\omega}}$ independently in Line 10. However, the number of selected asks may still exceed $q + m_\omega$. Therefore, we apply a $q + m_\omega + 1$-st auction in Line 14 and Line 15, where we choose the smallest $q + m_\omega$ asks, and set the payment $s$ as the $q + m_\omega + 1$-st smallest ask value. Till here, the algorithm selects no more than $q + m_\omega$ asks. If there are more than $q$ asks chosen, there will not be enough tasks to be allocated. To make sure that CRA allocates no more than $q$ tasks, we select $q$ asks randomly among these chosen asks with equal probability as final winning asks in Line 18 and all the others are losing asks. The corresponding payment is $s$ for each winning ask in Line 22.

In CRA, each ask bids for one task, whereas in RIT the ask from each user is in the format of $(t_j, k_j, a_j)$. Thus, we use Extract in Algorithm 2 to transform the ask vector $\mathbf{\alpha}$ into the vector $\alpha$ for each task type $\tau_i$, such that $\alpha$ contains all asks that bid for tasks in type $\tau_i$. Extract takes the task type $\tau_i$ and the ask vector $\mathbf{\alpha}$ as input, and outputs the ask vector $\alpha = (\alpha_1, \alpha_2, \ldots)$ for task type $\tau_i$ and a function $\lambda(\cdot)$, such that $\lambda(\omega) = j$ indicates that $\alpha_\omega$ comes from user $P_j$. For the ask $(t_j, k_j, a_j)$ from each user $P_j$, if $t_j$ equals to $\tau_i$, Extract expands its ask into $k_j$ asks of value $a_j$ in $\alpha$ and updates the function $\lambda(\cdot)$ in Line 6 of Algorithm 2. As an illustration, if $\mathcal{A} = ((\tau_1, 2, 3); (\tau_2, 3, 4); (\tau_1, 4, 2))$, after Extract$(\tau_1, \mathcal{A})$, the ask vector $\alpha = (3, 3, 2, 2, 2, 2)$, and the $\lambda$ function is computed as $\lambda(1) = 1, \lambda(2) = 1, \lambda(3) = 3, \lambda(4) = 3, \lambda(5) = 3, \lambda(6) = 3$.

Algorithm 1: CRA($\alpha, q, m_\omega$)

1) $x' \leftarrow 0$, $p' \leftarrow 0$;
2) Let $S$ be a sample of $\alpha$ where each ask is sampled with probability $\frac{1}{q + m_\omega}$ independently;
3) $s \leftarrow \min_{\alpha_\omega \in S} \alpha_\omega$;
4) Let $y$ be a uniform random value over $[0, 1]$;
5) $n_s \leftarrow \lceil z_s(\alpha) \rceil \{ 0 < z_s(\alpha) \}$, where $z_s(\alpha)$ is the number of asks at most $s$ in $\alpha$, and $\{ 0 < z_s(\alpha) \}$ is the nearest round-down value to $z_s(\alpha)$ in $\{ 2^i + y : z \in \mathbb{Z} \}$;
6) if $n_s \leq q + m_\omega$ then
7) Choose the smallest $n_s$ asks;
8) else
9) for each ask value $\alpha_\omega$ among the $n_s$ asks do
10) Choose $\alpha_\omega$ with probability $\frac{q + m_\omega}{2^{m_\omega}}$;
11) end
12) end
13) if there are more than $q + m_\omega$ asks chosen then
14) Choose the smallest $q + m_\omega$ asks as winning asks;
15) Let $s$ be the $q + m_\omega + 1$-st smallest ask value among the winning asks;
16) end
17) if there are more than $q$ asks chosen then
18) Randomly select $q$ asks as winners with equal probability and set the others as losers;
19) end
20) for each $\alpha_\omega$ in $\alpha$ do
21) if $\alpha_\omega$ is a winning ask then
22) $x'_\omega \leftarrow 1, p'_\omega \leftarrow s$;
23) end
24) end
25) return $(x', p')$.

Algorithm 2: Extract($\tau_i, \mathcal{A}$)

1) $\omega \leftarrow 0$;
2) for $j = 1, 2, \ldots, N$ do
3) if $t_j = \tau_i$ then
4) for $f = 1, 2, \ldots, k_j$ do
5) $\omega \leftarrow \omega + 1$;
6) $\alpha_\omega \leftarrow a_j, \lambda(\omega) \leftarrow j$;
7) end
8) end
9) end
10) return $(\alpha, \lambda(\cdot))$.

In Algorithm 3, RIT takes the job $\mathcal{J}$, users’ asks $\mathcal{A}$, the incentive tree $T$, and the probability $H$ as input, and outputs the indicator $\mathbf{x}$ and the final payment vector $p$. RIT consists of two phases, the auction phase and the payment
Algorithm 3: $\text{RIT}(\mathcal{J}, \mathcal{A}, T, H)$

/* Auction Phase */
1: $x \leftarrow 0$, $p \leftarrow 0$;
2: $p^{A} \leftarrow 0$, $\eta \leftarrow H^{\frac{1}{\rho}}$;
3: for each task type $\tau_i$ do
4: Define $k'$ as $((t'_1, k'_1, a'_1); (t'_2, k'_2, a'_2); \ldots)$;
5: $k' \leftarrow k'$;
6: $q \leftarrow m_i$, rounds $\leftarrow 0$;
7: \text{max} $\leftarrow \lfloor \log((1 - \frac{1}{q^c})K_{\text{max}} + \log(1 - \frac{2K_{\text{max}}}{q^c})) - e^{-\frac{q^c}{2}} \rfloor$;
8: while rounds $<$ max and $q > 0$ do
9: $(\alpha, \lambda(\cdot)) \leftarrow \text{Extract}(\tau_i, k')$;
10: $(x', p') \leftarrow \text{CRA}(\alpha, q, m_i)$;
11: for each ask $\alpha_s$ in $\alpha$ do
12: if $x'_s = 1$ then
13: $x_{\lambda(\cdot)(\cdot)} \leftarrow x_{\lambda(\cdot)(\cdot)} + 1$;
14: $p^{\alpha}_{\lambda(\cdot)(\cdot)} \leftarrow p^{\alpha}_{\lambda(\cdot)(\cdot)} + p'_{\lambda(\cdot)}$;
15: $k'_{\lambda(\cdot)(\cdot)} \leftarrow k'_{\lambda(\cdot)(\cdot)} - 1$;
16: $q \leftarrow q - 1$;
17: end
18: end
19: rounds $\leftarrow$ rounds $+ 1$;
20: end
21: /* Payment Determination Phase */
22: if all tasks in $\mathcal{J}$ are assigned then
23: for each user $j$ in the incentive tree do
24: $p_j = p_j^{A} + \sum_{t_i \in T_j \setminus t_j} \left(\frac{1}{2}\right)^t p^{A}_i$;
25: end
26: else
27: $x \leftarrow 0$, $p \leftarrow 0$;
28: end
29: return $(x, p)$.

determination phase. The auction phase (Line 2 to Line 21) applies multiple rounds of $\text{CRA}$s to allocate tasks and compute auction payments for each $\tau_i$. We first calculate the probability $\eta$ in Line 2 such that $\text{RIT}$ needs to be $(K_{\text{max}}, \eta)$-truthful for each type $\tau_i$. From Line 3 to Line 21, we allocate $m_i$ tasks and compute the corresponding auction payments. We set $q$ as the number of unallocated tasks and rounds as the number of finished rounds of $\text{CRA}$. We calculate $\text{max}$ for each $\tau_i$ as the maximum number of rounds to apply $\text{CRA}$ in Line 7. We prove it in Lemma 6.3 that if we run $\text{CRA}$ for no more than $\text{max}$ rounds for each $\tau_i$, $\text{RIT}$ is $(K_{\text{max}}, H)$-truthful. Before applying $\text{CRA}$ in Line 10, we first use $\text{Extract}$ to construct the ask vector $\alpha$ from the unallocated asks $k'$ in Line 9. After $\text{CRA}$, we update the indicator $x_{\lambda(\cdot)(\cdot)}$, the auction payment $p^{\alpha}_{\lambda(\cdot)(\cdot)}$, and the remaining capability $k'_{\lambda(\cdot)(\cdot)}$ for the user who submits a winning ask $\alpha_s$ in Line 13, Line 14, and Line 15, respectively. We also update $q$ and rounds in Line 16 and Line 19, respectively. The loop from Line 8 to Line 21 terminates when rounds exceeds max or all tasks in type $\tau_i$ have been allocated.

The payment determination phase (Line 22 to Line 28) computes the final payment for each user. If all tasks required by $\mathcal{J}$ are allocated, we apply the function in Line 24 to compute $p_j$ for each user $P_j$. Otherwise, $\mathcal{J}$ cannot be finished while satisfying the desired properties required by the design goal, and we set the payment for each user as 0 with no allocated tasks (Line 27). In Line 24, for each $P_j$, the payment $p_j$ sums up the auction payment $p_j^{A}$ and the weighted auction payments from its descendants whose task types are different from $t_j$. For each $P_i$ whose auction payment contributes to $p_j$, the corresponding weight is $(\frac{1}{2})^{i}$.

6. Desired Properties of $\text{RIT}$

In this section, we analyze $\text{RIT}$ and prove that $\text{RIT}$ satisfies the desired properties required by the design goal of this paper.

**Lemma 6.1:** The auction phase of $\text{RIT}$ is individually rational, i.e., if $a_j = c_j$, $P_j$'s utility from this ask is 0 since the payment would be 0 and there’s no cost incurred. If an ask $\alpha_s = c_j$ wins the auction, then the payment would be determined by Line 3 or Line 15 in Algorithm 1. If the payment is determined by Line 3, then all of the $n_s$ winning asks have ask values lower than the payment $s$ according to the definition of $n_s$ in Line 5, which means that $s \geq c_j$. If the payment is determined by Line 15, since $s$ is the $q + m_i + 1$-st smallest ask value and $\alpha_s$ is among the $q + m_i$ smallest ask values, we have $s \geq c_j$. Thus, for each $P_j$ and $\tau_i$, the auction payment $p_j^{A}$ equals to the sum of the payment $s$ in each round of $\text{CRA}$, and this value is no less than the total cost $x_j c_j$.

**Theorem 1:** $\text{RIT}$ is individually rational.

**Proof:** $U_j(t_j, k_j, c_j) = p_j - x_j c_j = \left(p_j^{A} - x_j c_j\right) + \sum_{t_i \in T_j \setminus t_j} \left(\frac{1}{2}\right)^t p^{A}_i$. By Lemma 6.1, we have $p_j^{A} \geq x_j c_j$. Since $p_j^{A} \geq 0$, we have $U_j(t_j, k_j, c_j) \geq 0$.

**Lemma 6.2:** $\text{CRA}$ is $k$-truthful with probability no less than $(1 - \frac{1}{q^c + m_i})^k + \log(1 - \frac{2K_{\text{max}}}{q^c}) - e^{-\frac{q^c}{2}}$.

**Proof:** Suppose there is a coalition of size $k$. Let $E_s$ be the event that an ask from the coalition is in the sample $S$ in Line 2 of Algorithm 1, let $E_c$ be the event that $n_s$ is not a $k$-consensus [12] and $n_s \geq q + m_i$, and let $E_s$ be the event that there are more than $q + m_i$ winners in Line 13 of Algorithm 1. By Lemma 15 of [13], if none of these events occurs, $\text{CRA}$ is $k$-truthful. Now we analyze the probabilities of these events:

- $Pr[ E_s ] = 1 - (1 - \frac{1}{q + m_i})^k$, since each ask is sampled independently with probability $\frac{1}{q + m_i}$.
- If $n_s \leq q + m_i$, $\text{CRA}$ is $k$-truthful when $E_s$ does not occur; if $n_s > q + m_i$, by Lemma 9 and Line 15 of [13], this probability is less than $-\log(1 - \frac{2K_{\text{max}}}{q^c + m_i})$. Thus, $Pr[ E_s \cup E_c ] \leq 1 - (1 - \frac{1}{q + m_i})^k - \log(1 - \frac{2K_{\text{max}}}{q^c + m_i})$.
- Since the expected number of winning asks in Line 13 is $n_s \times \frac{q + m_i}{2n_s} = \frac{q + m_i}{2}$, by Chernoff bound [25] and
Lemma 15 of [13], $Pr[E_0] \leq e^{-\frac{q+1}{8}}$.

Thus, CRA is $k$-truthful with probability $1 - Pr[E_c \cup E_s \cup E_o] \geq (1 - \frac{1}{m^k})^k + \log(1 - \frac{2k}{m^k}) - e^{-\frac{q+1}{8}}$.

\textbf{Remark 6.1:} For the lower bound of the probability in Lemma 6.2, it is easy to verify that it decreases with the decrement of $q$. Thus, when $q = 0$, the lower bound becomes $(1 - \frac{1}{m})^k + \log(1 - \frac{2k}{m^k}) - e^{-\frac{q}{8}}$. When the coalition size is relatively small compared to the job size, i.e., $\frac{k}{m} \ll 1$, the lower bound is close to one. Since each user can only have at most $K_{max}$ fake identities to form a coalition in CRA and $K_{max} \ll m_l$, CRA is $K_{max}$-truthful with a high probability. E.g., when $K_{max} = 10$ and $m_l = 1,000$, the lower bound is 0.98, which means that CRA is $(10,0.98)$-truthful.

However, we cannot directly apply the consensus technique from [12]. If we use the same technique, the new lower bound becomes $(1 - \frac{1}{m})^k + \log(1 - \frac{2k}{m^k}) - e^{-\frac{q}{8}}$ by a similar analysis of Lemma 6.2. After several rounds of CRAs in Algorithm 3, it is possible that there is a small value of $q$ such that $\frac{q}{8}$ is not close to zero, which drags down the lower bound of the probability. E.g., if $k = 10$ and $q = 50$, the new lower bound is 0.59, which is too low to be a satisfactory probability. To select $q + m$ winners in CRA, we need at least $2m$ tasks to bid for tasks in $\tau_i$ in Algorithm 2, as the number of unallocated tasks $q$ can be as most $m_l$ for each $\tau_i$. Thus, the incentive tree should propagate to $N$ users such that for each $\tau_i$, the users are able to complete at least $2m$ tasks.

\textbf{Lemma 6.3:} For all $P_j \in \mathcal{P}$, $\sum_{l=1}^{\delta(j)} U_{ji}(t_j, k_j, a_j) \geq \sum_{l=1}^{\delta(j)} U_{ji}(t_j, k_j, a_j)$ holds with probability at least $H$.

\textbf{Proof:} For each task type $\tau_i$, we apply at most $m_{max} = \left\lfloor \log((1 - \frac{1}{m})^{K_{max}} + \log(1 - \frac{2k}{m^k}) - e^{-\frac{q}{8}}) \right\rfloor$ rounds of CRAs. Each round of CRA is $K_{max}$-truthful with probability at least $(1 - \frac{1}{m})^{K_{max}} + \log(1 - \frac{2k}{m^k}) - e^{-\frac{q}{8}}$. To make all rounds of CRAs be $K_{max}$-truthful for $\tau_i$, it requires that each round of CRA is $K_{max}$-truthful, and this probability is no less than $(1 - \frac{1}{m})^{K_{max}} + \log(1 - \frac{2k}{m^k}) - e^{-\frac{q}{8}} = \eta^m \geq \eta$. Thus, the probability that the auction phase is truthful for all users is at least $\eta^m = H$.

In the payment determination phase, according to Line 24, $p_j = x_j c_j = p_j^A - x_j c_j + \sum_{P \in T_{ji}, t_i \neq t_j} \left(\frac{1}{2}\right)^{r_i} p_i^A$. $P_j$ has no influence on $\sum_{P \in T_{ji}, t_i \neq t_j} \left(\frac{1}{2}\right)^{r_i} p_i^A$ by changing its own ask value. Therefore, $p_j - x_j c_j$ is maximized if and only if $p_j^A - x_j c_j$ is maximized. Since the probability that $p_j^A - x_j c_j$ is maximized when revealing $c_j$ is at least $H$ for all $P_j \in \mathcal{P}$, the probability that the inequality holds is at least $H$.

\textbf{Lemma 6.4:} RIT is sybil-proof when the ask values of all $P_j$’s identities are the same, i.e., $U_{ji}(t_j, k_j, a_j) \geq \sum_{l=1}^{\delta(j)} U_{ji}(t_j, k_j, a_j)$, where $k_j = \sum_{i=1}^{\delta(j)} k_j$.

\textbf{Proof:} From Algorithm 3, we know that for each task type $\tau_i$, CRA is applied after Extract, which constructs the unit ask vector $\alpha$ from the ask vector $\mathcal{A}$. The number of fake identities does not influence the payment of the auction phase when $k_j = \sum_{i=1}^{\delta(j)} k_j$ and $a_j = a_j$ for each $P_j$, which indicates that $\sum_{i=1}^{\delta(j)} x_{ji} = x_j$ and the auction payment for each ask does not change. Since $U_{ji}(t_j, k_j, a_j) = p_j - x_j c_j$, $U_{ji}(t_j, k_j, a_j) = p_j - x_j c_j$, and $x_j = \sum_{i=1}^{\delta(j)} x_{ji}$, we need to prove that $p_j \geq \sum_{i=1}^{\delta(j)} p_i$ in order to prove this lemma.

Since an identity of $P_j$ is attached to either $P_j$’s parent or another identity of $P_j$ according to Section 3-B, for any sybil attack, we can imitate the attack by a series of “simpler” attacks, where in each of these “simpler” attacks, an identity of $P_j$ splits itself into two new identities. Therefore, if we can prove that none of these attacks could bring $P_j$ a higher payment, we can prove $p_j - x_j c_j \geq \sum_{i=1}^{\delta(j)} p_i - \sum_{i=1}^{\delta(j)} x_j c_j$.

![Fig. 4. An illustration of the attack when $P_{j_1}$ is the parent of $P_{j_2}$](image)

There are two kinds of sybil attacks if $P_j$ splits one of its identities $P_{j_2}$ into two new identities, $P_{j_2}$ and $P_{j_2}$. The first is that $P_{j_1}$ is the parent of $P_{j_2}$, as illustrated in Fig. 4. Let $z_i$ be the number of $P_j$’s identities where $P_j$ is a descendant of these identities. Let $r_i$ be the distance from a user $P_i$ to the root after the attack. Let $T_{j_2}$ denote the set of users who are descendants of $P_{j_2}$. The auction payment for each user does not change after the attack. If $P_i$ is not a descendant of $P_{j_2}$, $r_i = r_i$; otherwise, $r_i = r_i + 1$. Thus, the change of $P_j$’s payment from this sybil attack is $\sum_{P_i \in T_{j_2}, t \neq t_j} [(z_i + 1)\left(\frac{1}{2}\right)^{r_i} p_i^A - z_i\left(\frac{1}{2}\right)^{r_i} p_i^A] = \sum_{P_i \in T_{j_2}, t \neq t_j} \frac{z_i}{2} \left(\frac{1}{2}\right)^{r_i} p_i^A$. Since $z_i \geq 1$, this change of payment is non-positive, which indicates that the first kind of attack could not increase $P_j$’s utility.

![Fig. 5. An illustration of the attack when $P_{j_1}$ and $P_{j_2}$ are siblings](image)

The second kind of attack is that $P_{j_1}$ and $P_{j_2}$ are siblings and their parents are the parent of $P_{j_3}$, as illustrated in Fig. 5. The total auction payment of all identities of $P_j$ does not change. The auction payments for all other users do not change. $r_i = r_i$ for each user $P_i$. Thus, the utility of $P_j$ does not change after the sybil attack.
Therefore, the sybil attack with the same ask value could not bring \( P_i \) a higher utility.

**Theorem 2:** \( \text{RIT} \) is truthful and sybil-proof with probability at least \( H \), i.e., \( U_j(t_j,K_j,c_j) \geq \sum_{i=1}^{\delta(j)} U_j(t_j,k_{ij},a_{ij}) \) with probability at least \( H \), where \( \sum_{i=1}^{\delta(j)} k_{ij} \leq K_j \).

**Proof:** From Lemma 6.3, we know that \( \sum_{i=1}^{\delta(j)} U_j(t_j,k_{ij},c_j) \geq \sum_{i=1}^{\delta(j)} U_j(t_j,k_{ij},a_{ij}) \) for any \( P_j \) with probability at least \( H \). From Lemma 6.4, we have \( \sum_{i=1}^{\delta(j)} U_j(t_j,k_{ij},c_j) \leq U_j(t_j,\sum_{i=1}^{\delta(j)} k_{ij},c_j) \). Suppose that there is a user \( P_j \) whose parent is an identity of \( P_j \) with \( t_j' = t_j \), \( K_j' = K_j = \sum_{i=1}^{\delta(j)} k_{ij} \), and \( c_j' = c_j \). Since \( \text{RIT} \) is individually rational, we have \( U_j(t_j,K_j,c_j) \geq U_j(t_j,\sum_{i=1}^{\delta(j)} k_{ij},c_j) + U_j(t_j,K_j',c_j) \). If we take \( P_j \) as another fake identity of \( P_j \), by Lemma 6.4 we have \( U_j(t_j,\sum_{i=1}^{\delta(j)} k_{ij},c_j) + U_j(t_j,K_j',c_j) \leq U_j(t_j,\sum_{i=1}^{\delta(j)} k_{ij} + K_j',c_j) = U_j(t_j,K_j,c_j) \). Thus, we have \( U_j(t_j,K_j,c_j) \geq \sum_{i=1}^{\delta(j)} U_j(t_j,k_{ij},a_{ij}) \) with probability at least \( H \) for any \( P_j \).

**Theorem 3:** \( \text{RIT} \) is computationally efficient.

**Proof:** First we analyze the time complexity of the auction phase. For each task type \( \tau_i \), we run at most \( \frac{\lg n}{\ln(1 - \frac{1}{m})^{K_{max}} + \log(1 - 2K_{max}^{m_i})} \) rounds of CRAs.

Since \( (1 - \frac{1}{m})^{K_{max}} + \log(1 - 2K_{max}^{m_i}) \approx e^{-\frac{2K_{max}^{m_i}}{m_i}} \), when \( K_{max}^{m_i} \ll 1 \), we have \( (1 - \frac{1}{m})^{K_{max}} + \log(1 - 2K_{max}^{m_i}) - e^{-\frac{2K_{max}^{m_i}}{m_i}} \leq e^{-\frac{2K_{max}^{m_i}}{m_i}} \). Thus, the number of rounds of CRAs is \( O(\sum_{i=1}^{m} - \lg n \times m_i) \in O(\frac{|J|}{K_{max}}) \). For each CRA, the running time is \( O(\sum_{j=1}^{\delta(j)} k_{ij}) = O(NK_{max}) \). Each Extract is also bounded by \( O(NK_{max}) \). Thus, the time complexity for the auction phase is \( O(\frac{|J|}{K_{max}} \times N) = O(N|J|) \).

For the payment determination phase, to calculate the payment for each user \( P_j \), we sum up the weighted auction payments from the users in \( T_j \). By computing this payment recursively from leaf to root, the time complexity of this phase is \( O(N) \). Thus, the time complexity of \( \text{RIT} \) is \( O(N|J|) \), which indicates that \( \text{RIT} \) is computationally efficient.

**Theorem 4:** \( \text{RIT} \) satisfies solicitation incentive.

**Proof:** Consider that there is a user \( P_i \) joining the incentive tree. Let \( P_j^A \) be the new auction payment of \( P_j \). If \( t_j = t_j \), no matter where \( P_i \) joins in the incentive tree, the change of \( P_j \)'s utility is \( P_j^A - P_j^A \). If \( t_i \neq t_j \), when \( P_j \) joins as a descendant of \( P_j \), the change of \( P_j \)'s utility is \((1/2)^{\tau_i} P_j^A \geq 0 \). To make this increment of utility as large as possible, \( P_j \) would like \( P_i \) to join as its child. When \( P_j \) joins as a non-descendant of \( P_j \), \( P_j \)'s utility does not change. Thus, comparing with \( P_i \) joining as a child of another user, \( P_j \) prefers \( P_i \) to joining as its own child for a higher utility, which indicates that \( \text{RIT} \) satisfies solicitation incentive.

## 7. Performance Evaluation

In this section, we implemented \( \text{RIT} \) and present the extensive performance evaluation results.

### A. Simulation Setup

We present the setup of the simulation as follows. The job \( J \) consists of tasks in \( m = 10 \) task types. Each user \( P_j \)'s task type \( t_j \) is randomly distributed among the 10 task types, and \( k_j \) is uniformly distributed over \((0, 20]\). The ask value \( a_j \) is uniformly distributed over \((0, 10]\). We set the threshold probability \( H = 0.8 \). All evaluations were tested on a Linux system with 3.4 GHz Core i-7 CPU and 8GB memory. All results are averaged over 1000 times.

To build the incentive tree, we used the data from [21], a twitter social network with over 80,000 users. A twitter user \( P_i \) follows user \( P_j \) indicates that \( P_j \) has an influence on \( P_i \). Thus, \( P_i \) may join the incentive tree as a child of \( P_j \). We generate a spanning forest of the social network where each user refers all of its un-joined neighbors into the incentive tree. We set the platform as the root of the incentive tree and attach all roots of the spanning forest as the children of the root. If multiple invitations arrive at a user at the same time, we break the ties by choosing the one with the smallest index among the inviters as the parent. E.g., if \( P_i \) receives invitation from \( P_1 \) and \( P_2 \) at the same time, we add \( P_6 \) as \( P_i \)'s child.

### B. Performance Metrics

During the simulation, we study average user utility, total payment, running time, and dishonest user utility as the metrics to evaluate \( \text{RIT} \). We study the impact of the payment determination phase on the performance metrics introduced above, by comparing the results of the auction phase with the final results of \( \text{RIT} \).

To evaluate the impact of the number of users on average user utility, total payment, and running time, we set \( m = 5000 \) for each \( \tau_i \) and varied the number of users from 40000 to 80000 with an increment of 1000. The results are presented in Fig. 6(a), Fig. 7(a), and Fig. 8(a), respectively.

To evaluate the impact of the number of tasks on average user utility, total payment, and running time, we fixed \( n = 30000 \), and varied \( m_i \) from 1000 to 3000 with an increment of 100 for each task type \( \tau_i \). The results are presented in Fig. 6(b), Fig. 7(b), and Fig. 8(b), respectively.

To evaluate the properties of sybil-proofness and truthfulness, we fixed \( n = 10000 \) and let \( m_i \) be uniformly distributed over \((100, 500]\) for each task type \( \tau_i \). We randomly picked a user \( P_{29} \) whose auction payment is non-zero when asking its true cost \( c_{29} = 5.5 \). We also have \( K_{29} = 17 \), which means that \( P_{29} \) cannot generate more than 17 identities. We varied the number of identities of \( P_{29} \) from 2 to 17, and let \( P_{29} \) randomly generate the identities. We monitored the total utility of \( P_{29} \) from its dishonest behavior. Further, we picked three
ask values for $P_{29}$’s identities and monitored the corresponding
utilities. The three ask values are $a_{29} = c_{29} = 5.5$, $a_{29} = 6.5$, and $a_{29} = 6.25$. We present the results in Fig. 9.

C. Simulation Results

In this subsection, we present our simulation results on RIT
and provide some analysis on these results.

From Fig. 6(a) we observe that with more users, the average
user utility decreases. This is because with more users, the
competition among the users becomes more fierce and as a
result, the average auction payment decreases, which leads
to a decrement to the average user utility. In Fig. 6(b),
when the number of users are fixed, the average user utility
increases with the increment of the job size. This is because
with more tasks to be allocated, the average auction payment
increases, which leads to the increment of average user utility.
Furthermore, by adding the payment determination phase to
the auction phase, the average auction payment decreases. However, with more users, the
average auction payment increases since each user gets rewards for its solicitation, which confirms
our analysis of solicitation incentive and demonstrates that RIT incentivizes users for solicitation.

From Fig. 7(a) it is observed that the total payment of the
platform does not increase remarkably with the increment
of the number of users. The reason is that despite the increase
of the number of users, the number of allocated tasks is fixed. Besides, with the increment of users, the average
auction payment decreases. However, with more users, the payment for solicitation of each user may increase. Combining
these factors, the total payment of the platform does not increase remarkably. In Fig. 7(b), the total payment of the
platform increases with the job size. This is because with more tasks, the number of winning users increases, and the
payments from RIT increase as well, which leads to the increment of the total payment. Furthermore, we observe that by applying the payment determination phase to the auction phase, the total payment increases. The increment is no more than the total auction payment, i.e., $\sum_{j \in P_i} (p_j - p_j^A) \leq \sum_{j \in P_i} r_j (\frac{1}{2})^r p_j^A \leq \sum_{j \in P_i} p_j^A$, where $r_j \geq 1$ for each user $P_j$. This indicates that by applying the incentive tree based payment determination phase, the platform pays no more than the total auction payment for solicitation.

In Fig. 8(a) and Fig. 8(b), we observe that the running
time is increasing in an approximately linear speed with the
size of the users and the job size, respectively, which backs up the analysis of computational efficiency in Lemma 3.
Furthermore, by applying the payment determination phase,
the time complexity of the algorithm still grows linearly.

In Fig. 9, we observe that with the increment of the number of identities, the utility of $P_{29}$ decreases, which is a demonstration of the sybil-proofness of RIT. Apart from revealing its cost $c_29 = 5.5$, we also evaluate the utility of $P_{29}$ when it asks values deviating from its true cost by having $a_{29} = 6.5$ and $a_{29} = 6.225$. We observe that when $a_{29} = c_{29}$, $P_{29}$’s utility is larger than those of the rest of the ask values, which further confirms the truthfulness of RIT.

8. Conclusions

In this paper, we designed a robust crowdsensing incentive
mechanism RIT, which is an auction-based incentive tree
mechanism, to motivate users for participation and solicitation. We proved that RIT is truthful and sybil-proof with probability
at least $H$, for any given $H \in (0, 1)$. We also proved that RIT satisfies individual rationality, computational efficiency, and solicitation incentive. We implemented RIT and the performance evaluation results confirm our analysis.