Game Theoretical Analysis of Coexistence in MIMO-Empowered Cognitive Radio Networks

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***Abstract*—In Cognitive Radio Networks (CRNs), the spectrum underlay approach enables primary and secondary networks to transmit simultaneously, as long as the interference from the secondary network to the primary network is below certain threshold. As the recent advancement of the underlay approach, the transparent coexistence approach exploiting MIMO interference cancellation is proposed. Previous works assume that the secondary networks will completely follow the spectrum access rules by controlling their interference to the primary network. However, this may not always hold in practice due to the selfish nature of CRN users. In this work, we study the multi-hop MIMO-empowered secondary network's incentives of following or violating this rule through compliantly canceling using MIMO or non-compliantly ignoring its interferences towards the primary network. Specifically, we model the coexistence between the primary and secondary networks as a Stackelberg game. By analyzing and comparing the equilibriums, we obtain several insights that reveal the incentive for the secondary network to be non-compliant and the methodology to deal with such type of selfish secondary networks.**

I. INTRODUCTION

In cognitive radio networks (CRNs) [1], primary and secondary networks share the spectrum jointly. According to the spectrum access rules imposed by FCC, primary network has the priority in spectrum accessing while the secondary networks could transmit simultaneously as long as the primary network is not interfered with. Multiple previous works [2] have been proposed to exploit the limited spectrum resources in order to maximize the coexisting networks' throughputs in CRN domain under this rule by using traditional underlay, overlay, over interweave paradigm [28]. As the recent advancement of underlay paradigm, 'transparent coexistence' [26] improves the coexistence performance in multi-hop CRNs by utilizing MIMO interference cancellation (IC) [2] to perform concurrent transmission.

However, these works didn't capture the 'selfish' nature of the coexistence problem, where the secondary network might maximize its own throughput without controlling its interference towards the primary network. Using 'transparent coexistence' as example, it assumes the secondary networks cooperatively mitigate the interferences using both transmitter-side and receiver-side IC, but it might not have the incentive to mitigate its interference using transmitter-side IC to the primary network, since doing so consumes its own DoF resources that could be otherwise used to transmit more streams.

To deal with such selfish secondary networks and to guarantee the primary network's service quality, spectrum coexistence rule enforcement schemes [3] [9] were proposed. However, the effectiveness of rule enforcement could be limited in practice as it is impossible to monitor the unlimited physical wireless domain. In some areas where enforcement entities are not present, some unruly secondary network devices might selfishly maximize its own throughput regardless of its interference to the primary network.

To handle the interferences from selfish secondary networks, the primary network needs to come up with a solution, which doesn't rely on rule enforcement. Specifically, the primary network needs to take the secondary network's 'selfish' nature into account before choosing its own transmitting scheme in terms of the number DoFs used for spatial multiplexing. It needs to spare enough number of DoFs to deal with the interference from 'selfish' secondary network by using the receiver-side IC. The remaining question is how many DoFs is needed to deal with such selfish secondary networks. To answer this question, we rely on game theoretical approach.

Several works [20] [25] have applied game-theoretical approach to study the coexistence problem in CRNs with MIMO. However, these game-theoretical works didn't consider the multi-hop-network cases, which is a common form of secondary networks. The main reason is that they applied a traditional SNR model, which is precise but non-tractable in multi-hop cases. In this work, we choose the widely used DoF model, which is a close approximation of SNR model under the assumption of high SNR. In addition, they studied the competition among different secondary networks, each still following the spectrum access rule by mitigating their interferences to the primary network. In our work, we study the game between the primary and secondary networks, and analyze the secondary network's selfish incentives.

In this work, we use a game-theoretical approach to study the coexistence between the primary network and a multi-hop secondary network, where each node has wireless MIMO capability. We consider two types of secondary networks: selfish-compliant (cooperative) or selfish-non-compliant (non-cooperative). Each network has MIMO capability to perform spatial multiplexing (SM) and interference cancellation (IC). Our work is the extension of [10], in which we studied the coexistence problem of two single-link networks. Compared with the previous work, we extend to a more general case where the secondary network could be multi-hop and multi-flow. The major challenges are two-fold: 1) the intricacy of joint link scheduling and DoF allocations in multi-hop networks makes it difficult to derive the secondary network's optimal response. 2) Unlike the single-link case, we find it is hard to derive a closed-form expression of leader's utility as a function of leader's strategy and follower's strategy.

To solve these challenges, we first formulate a mix-integer-linear-programming problem to derive the multi-hop secondary network's optimal response. Second, we design an algorithm to derive optimal primary network's strategy through feasibility checking. From the theoretical and numerical results, we obtain several insights of the secondary network's selfish incentives and the methodology to deal with it. The major contributions of our work are: 1) we are the first to study the coexistence between a primary network and a *multi-hop, multi-flow* secondary network using game-theoretical approach; 2) we are the first to study the incentive of secondary network's selfish spectrum access behaviors in the MIMO empowered CRN domain.

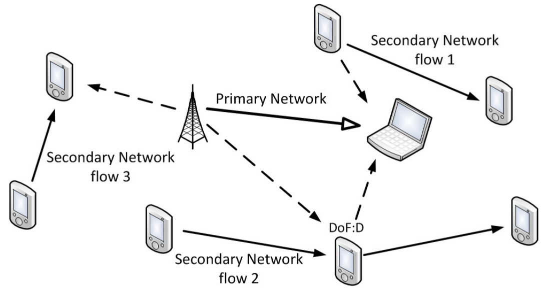

Fig. 1: Coexistence example: a single-link primary network coexists with a multi-hop and multi-flow secondary network

This paper is organized as follows: In Section II, related work is presented. Section III introduces the system model. In Section IV, we propose the frameworks for analyzing the coexistence game between the primary network and the multi-hop secondary network. The numerical results are shown in Section V. Section VI concludes the paper.

II. RELATED WORK

Cooperative Interference Avoidance and Cancellation Traditional approach to the coexistence problem in general wireless networks is based on Interference Avoidance (IA), which separates the transmission in temporal, spatial, or frequency domain [13], [15], [21]. These IA works are all based on traditional single antenna, while our work studies the general MIMO network with multiple antennas on each device and the single-antenna network is only a special case. Recent advances in IC enables the interference signal mitigation even within the same frequency band, which could enhance standalone or coexisting-network's throughput [4], [5], [8], [12], [23]. All these works on IA and IC utilizing MIMO assume the networks compliantly avoid/cancel the interferences to all other networks by following the predefined spectrum access rule or protocols. In this work, we challenge this assumption by studying the secondary network's incentive in CRNs of following/violating the spectrum access rule.

Cooperative Spectrum Sharing Manna et.al. [17] studied the cooperative spectrum sharing within a single-link coexisting scenario. They proposed an overlay sharing scheme and showed its gain for both primary and secondary networks. Duan et.al. [6] studied the cooperative spectrum share problem using contract theory where the primary network offers contracts and the secondary users select a contract which gives them spectrum access opportunities by offering its relaying power. Xu et.al. [26] [27] utilized wireless MIMO to enhance throughput in CRNs through cooperative interference cancellation. However, all these works assume the secondary network either relay packets for primary network or mitigate interference to it cooperatively, while we try to unveil the selfish incentive of secondary network by modeling the coexisting problem as a non-cooperative game.

Game Theoretical Analysis in CRNs Saad et.al. [18] studied the coexistence in CRNs using game-theoretical approach. However, they focused on the spectrum-sensing problem by modeling it as a coalition game aiming at increasing the detection probability of primary network's signal. Jiang et.al. [14] [19] studied the joint spectrum sensing and access problem. However, their views are still very different from ours as they study the spectrum sensing and access game among only the secondary-network users. In addition, in our work we don't consider the spectrum-sensing problem and assume the secondary network could always detect the primary network's presence through perfect spectrum sensing. Gong [7] studied the spectrum access problem in CRN though a power-control game but without considering MIMO. In addition they also focus on the game among secondary networks. Wang et.al. [24] studied the spectrum share and relaying power for both networks in the cooperative spectrum sharing problem. Though they assume the secondary network is selfish against the primary user, it still follows the spectrum-share rule defined by the primary network. Scutari and Wang et.al [20] [25] studied the spectrum access problem in CRNs with MIMO. Similar to our work, they modeled their problem as a non-cooperative game, and derived the optimal precoding strategies of secondary-network users in presence of other secondary network users. However, the difference with our work is that they focus on the single-link case, while we are capable of handling the more-general multi-hop secondary network case. The reason is that we choose a simplified DoF model, which is tractable for the multi-hop analysis compared with their SNR model. In addition, same as previous works, their works also focus on the coexistence game among multiple secondary network users, while we aim at studying the selfish incentives of secondary network against the primary network.

III. SYSTEM MODEL

We assume a single-link primary network $\mathcal{N}_p(\mathcal{V}_p, \mathcal{E}_p)$ coexists with a general multi-hop secondary network $\mathcal{N}_s(\mathcal{V}_s, \mathcal{E}_s)$ with multiple flows. Each network has MIMO capability [16], using which each device could perform IC to mitigate interference from/to any other device and SM to transmit multiple streams concurrently. Each network is considered as a player, and its nodes are coordinated by a central controller which determines what strategy to adopt. Both controllers have the complete information about their own network topologies respectively. In addition the primary network's controller also has the complete selfish-type, topology and flow information of the secondary network. The interference graph is assumed known by both networks. Both networks coexist within a single frequency band. We assume the primary and secondary networks could arbitrarily select their strategies in terms of the number of spatial-multiplexing DoFs and interference-cancellation DoFs. As multi-hop network is considered, we divide time into a number of discrete time slots $t \in \{1, 2..T\}$. However the primary network is not as agile as secondary network, i.e., it is not capable of swiftly changing its strategy in each time slot.

$s_p, \boldsymbol{s}_s$	strategy of primary and secondary network
$\theta_{i,j}(t), \pi_i(t)$	relative and global cancellation ordering on nodes i, j
$z_l(t)$	number of streams sent on link l in time slot t
u_p, u_s	primary and secondary network's utility

TABLE I: Major Notations

We assume that the primary network always accesses the spectrum first by freely selecting its accessing strategy, and the secondary network determines its spectrum access responsive strategy afterwards. Two types of secondary networks are studied, which are 1) type-1: selfish-compliant network meaning that the secondary network aims at maximizing its throughput without generating interference to the primary network, and 2) type-2: selfish-non-compliant network meaning that the secondary network maximizes its own throughput regardless of its interference to the primary network. We assume each network's device could have arbitrary number of antennas.

IV. GAME THEORETICAL ANALYSIS

In CRNs, the primary network always has the priority in spectrum access. Therefore our game is formulated as a Stackelberg game, in which the primary network makes spectrum access decision first, and the secondary network makes its decision afterwards. Each network tries to maximize its throughput. We will study a general multi-hop case, in which the primary network coexists with the secondary network, in the type of either selfish-compliant or selfish-non-compliant.

The multi-hop secondary-network case is more complicated than the single-link case. This is mainly due to the intricacy of link scheduling in multi-hop networks. We will show that though we can't express the optimal response strategy in closed-form, we can still derive the optimal response strategy of the secondary network through solving a mixed integer linear programming (MILP) problem. Based on the optimal response, we can further derive the optimal leading strategy of the primary network through an algorithm, in which we calculate the interference degree generated by the secondary network and then check each primary network strategy's feasibility.

A. Strategy Space

For the single-link primary network, the strategy $s_p = z_{l_p}$ denotes the number of DoFs spent on transmitting streams concurrently on link l_p, which should be bounded by its antenna number $min(A_{p_t}, A_{p_r})$. We assume the primary network's receiver uses the remaining DoFs for receiver-side IC to mitigate the possible interferences back from secondary network.

For the secondary network, the strategy $\boldsymbol{s}_s = [\boldsymbol{z}_{l_0} ... \boldsymbol{z}_{l_L}]$, where $\boldsymbol{z}_{l_k} = [z_{l_k}(0), ... z_{l_k}(T)], k \in \{0, ... L\}$ denotes the number of streams transmitted in each time slot $t \in \{0, ... T\}$ on each link l_k. The remaining DoFs are used for performing interference cancellation at both the transmitter and receiver sides. Similarly, the secondary network's strategy should also be bounded by the antenna number on each of its devices. In addition, it should also satisfy several intra-network constraints [22] including half-duplex, node ordering, transmitter-side and receiver-side IC constraints to guarantee the strategy's feasibility:

$$x_i(t) + y_i(t) \leq 1 \qquad (i \in \mathcal{V}_s, 1 \leq t \leq T) \tag{1}$$

$$\pi_i(t) - N \cdot \theta_{j,i}(t) + 1 \leq \pi_j(t) \leq \pi_i(t) - N \cdot \theta_{j,i}(t) + N - 1, \quad (i, j \in \mathcal{V}_s, 1 \leq t \leq T) \tag{2}$$

$$x_i(t) \leq \sum_{l \in \mathcal{L}_{i,out}} z_l(t) + [\sum_{j \in \mathcal{I}_i,} (\theta_{j,i}(t) \sum_{k \in \mathcal{L}_{j,in}}^{Tx(k) \neq i} z_k(t))] x_i(t) \leq A_i x_i(t), \quad (i \in \mathcal{V}_s, 1 \leq t \leq T) \tag{3}$$

$$y_i(t) \leq \sum_{l \in \mathcal{L}_{i,in}} z_l(t) + [\sum_{j \in \mathcal{I}_i} (\theta_{j,i}(t) \sum_{k \in \mathcal{L}_{j,out}}^{Rx(k) \neq i} z_k(t))] y_i(t) \leq A_i y_i(t), \quad (i \in \mathcal{V}_s, 1 \leq t \leq T) \tag{4}$$

The variables $x_i(t), y_i(t)$ denote whether node i sends or receives at time slot t. $z_l(t)$ denotes the number of streams sent on link l. $\theta_{i,j}$ denotes the cancellation order between node i and j. $\pi_i(t)$ denotes the global cancellation order of node i. A_i is the antenna number parameter at node i. $\mathcal{I}_i$ denotes the interference set of node i. $\mathcal{L}_{i,in}$ and $\mathcal{L}_{i,out}$ denote node i's inward and outward link sets. The constraint (1) requires each node to be half-duplex thus can't send and receive simultaneously. The node ordering constraint in (2) is used to establish a cancellation order for each pair of nodes, which ensures the feasibility of the stream transmitting strategy $\boldsymbol{s}_s$. Constraints (3)(4) guarantee that the overall number of DoFs used for spatial multiplexing and interference cancellation at each node should be bounded by its antenna number. Through these constraints, we can guarantee the secondary network stream-transmitting strategy's feasibility inside its network.

B. Utility

The utility u_p and u_s is the total number of streams successfully transmitted on all flows in all time slots for the primary and secondary networks respectively. When calculating the utilities, we should take the other network's strategies into account, as these strategies could generate external interferences. For the secondary network, as it could have multiple flows $f \in \mathcal{F}_s$ each with multi-hops where $\mathcal{F}_s$ denotes the set of flows in secondary network, its utility is defined as the summation of all flows' rates $r(f)$ which is the time-average throughput of this flow. We denote its utility as $u_s = \sum_{f \in \mathcal{F}_s} r(f)$. We will show how to derive its utility under the interferences from primary network in next subsection. For the leading primary network, its utility is defined as:

$$u_p(\boldsymbol{s}_s, s_p) = \begin{cases} s_p & \text{if } s_p + \mu_{r_p}(\boldsymbol{s}_s) < A_{r_p} \\ 0 & \text{if } s_p + \mu_{r_p}(\boldsymbol{s}_s) > A_{r_p} \end{cases} \tag{5}$$

where $\mu_{r_p}(\boldsymbol{s}_s)$ denotes the maximum total number of interference received back from the secondary network. (5) denotes that the primary network's utility is 0 if it is interfered with by the secondary network. This is because we assume primary network's QoS should be always guaranteed.

C. Equilibrium

We will first analyze the optimal response strategy of the secondary network. After that we will derive the optimal leading strategy of the primary network and finally the equilibrium.

Type-1 Multi-hop Secondary Network

When dealing with the type-1 secondary network, the primary network's optimal strategy is shown in lemma 4.1.

Lemma 4.1: $s_p^* = min(A_{t_p}, A_{r_p})$

Proof: The proof is very straightforward, as the primary network knows the complete information of the secondary network's compliant behaviors, it should choose to transmit as many streams as it could. ■

For the optimal response, given any primary network's strategy, the secondary network will choose its strategy without generating interference to the primary network. Different from the single-link case, it is more difficult to derive the closed-form expression of the optimal response. However, we show that the optimal response could be derived through solving a MILP problem (ORT1), which is very similar to that in [26]. To use their optimization model, we need to set $z_{l_p}(t) = s_p^*, \forall t$. The optimization solution and result are the secondary network's optimal response and utility respectively against primary network's optimal strategy.

Lemma 4.2: The solution of problem ORT1 is the optimal response of type-1 secondary network $\boldsymbol{s}_s^*(s_p^*)$ against primary network's optimal strategy s_p^*.

Proof: According to the model in [26], the secondary network takes the primary network's transmitted streams into its transmitter-side constraint, thus the primary network is not interfered with. ■

Theorem 4.3: $(s_p^*, \boldsymbol{s}_s^*(s_p^*))$ is the equilibrium of the type-1 secondary-network follower case, where s_p^* and $\boldsymbol{s}_s^*(s_p^*)$ are defined in Lemma. 4.1 and Lemma. 4.2 respectively.

Type-2 Multi-hop Secondary Network

The type-2 secondary network follows no predefined rule, thus it doesn't manage its interference towards the primary network. We first need to establish the model of our coexistence problem. The first one is primary network's strategy. Similar as defined in the type-1 case, we use z_{l_p} to denote the number of streams sent on primary link l_p:

$$z_{l_p}(t) = s_p, \quad \forall t \tag{6}$$

The second one is the canceling order constraint, in which we let all nodes in secondary network $\mathcal{N}_s$ to cancel the interferences from primary network's transmitter p_t.

$$\theta_{p_t,i} = 1, \quad \forall i \in \mathcal{N}_s \tag{7}$$

The secondary network also needs to satisfy all the constraints in (1) (2) (3) to guarantee the solution feasibility. In addition, the secondary network needs to cancel the interference generated by primary network's strategy s_p by adding it into its receiver-side constraint:

$$\max\ u_s = \sum_{f \in \mathcal{F}_s} r(f)$$

s.t.

Primary network strategy constraint(6)
Cancelling ordring constraint(7)
Half duplex constraint(1)
Node ordering constraints(2)
Tx DoF constraints: no IC to primary netwok(3)
Rx DoF constraints (8)
Link capacity model(9)
Flow rate $\leq$ link capacity(10)

Fig. 2: Optimal response of the secondary network (ORS2(s_p)) given primary network's strategy

$$y_i(t) \leq \sum_{l \in \mathcal{L}_{i,in}} z_l(t) + [\sum_{j \in \mathcal{I}_i} (\theta_{j,i}(t) \sum_{k \in \mathcal{L}_{j,out}}^{Rx(k) \neq i} z_k(t)) + \theta_{p_t,i}(t) z_{l_p}(t)] y_i(t) \leq A_i y_i(t), \quad (i \in \mathcal{V}_s, 1 \leq t \leq T) \tag{8}$$

Note that we don't change the transmitter-side constraint in (3) as the secondary network is selfish-non-compliant thus it doesn't control its interference to the primary network. We have link capacity constraint (9) and flow-rate constraint (10) to calculate the secondary network's throughput utility. c_l denotes link l's capacity. $r(f)$ denotes rate on flow f.

$$c_l = \frac{1}{T} \sum_{t=1}^{T} z_l(t), \quad (\forall l \in \mathcal{L}_s, 1 \leq t \leq T) \tag{9}$$

$$r(f) \leqslant c_l \quad (\forall l \in f, f \in \mathcal{F}_s) \tag{10}$$

To derive the equilibrium, we start from secondary network's optimal-response strategy. The definition of the ORS2 problem is shown in Fig. 2.

Lemma 4.4: Solution of problem ORS2(s_p) is the type-2 secondary network's optimal response strategy $\boldsymbol{s}_s^*(s_p)$ under arbitrary primary network's strategy s_p. The objective value is the secondary network's maximum utility u_s^* under $\boldsymbol{s}_s^*(s_p)$.

Proof: The canceling-order constraint (7) and transmitter-side constraint (3) guarantees the secondary network doesn't take the primary network's receiver into account when performing transmitter-side IC. Therefore the response is selfish-non-compliant. ■

We have shown that the optimal response of the secondary network could be derived through solving a mixed-integer-nonlinear-programming (MINLP) problem shown in Fig. 2. Through comparing the formulations of ORS2 and ORS1, we could find that in ORS2, the strategy space is loosened as no transmitter-side IC is considered towards the primary network. We use reformulation linearization technique (RLT) [26] to reformulate our problem into a mixed-integer-linear-programming (MILP) problem, which is NP-hard in general. The details of the reformulation can be found in our technical report [11]. It is not presented here due to space limitation. To

Algorithm 1 Calculating primary network's utility under secondary network's strategy

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input: Network topology $G$, secondary network's strategy $\boldsymbol{s}_s$; primary network's strategy $s_p$
output: Primary network's utility $u_p$
for $t = 1 : T$ do
    $\mu_{r_p,t} = 0$ , $\mu$ denotes the interferences received by a node
    for all $i \in \mathcal{I}_{r_p}$ do  $\mu_{r_p,t} += z_{l_{k:tx=i}}(t)$, $\mathcal{I}$ denotes the link sets that cause interference
    end for
end for
$\mu_{r_p}(\boldsymbol{s}_s) = \max_t\{\mu_{r_p,t}\}$
if $s_p + \mu_{r_p}(\boldsymbol{s}_s) \leqslant A_{r_p}$ then $u_p(\boldsymbol{s}_s, s_p) = s_p$
else  $u_p(\boldsymbol{s}_s, s_p) = 0$
end if
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Algorithm 2 Finding primary network's optimal strategy

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input: Network topology $G$
output: Primary network's optimal strategy $s_p^*$, and corresponding optimal utility $u_p^*$
for $s_p = 1 : min(A_{t_p}, A_{r_p})$ do
    Solve problem ORS2($s_p$), record solution $\boldsymbol{s}_s^*(s_p)$
    Run Alg.1 with input $\boldsymbol{s}_s^*(s_p)$ , $s_p$
    Collect $u_p(\boldsymbol{s}_s^*(s_p), s_p)$
end for
$s_p^* : s.t. u_p^* = \max_{s_p}\{u_p(\boldsymbol{s_s}^*(s_p), s_p)\}$
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solve this problem, we choose IBM's CPLEX solver, which is based on branch and bound technique. We will leave the more-efficient approximation approach in future work. The remaining problem is how to derive the optimal primary network's strategy. To derive primary network's optimal strategy, we first need to know the primary network's utility under secondary network's interference in return, which could be derived by Alg. 1. Using Alg. 1 as a building block, we derive the optimal leader's strategy through Alg. 2.

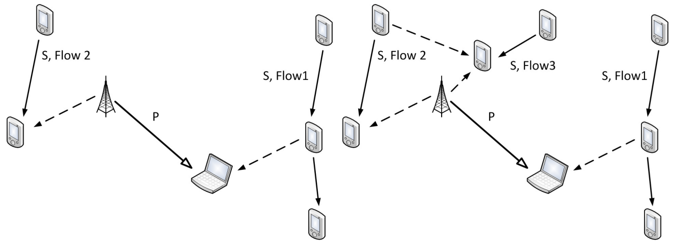
Lemma 4.5: The output of Alg. 2 is the primary network's optimal strategy s_p^* when coexisting with a type-2 multi-hop secondary network.

Proof: In Alg. 2, each primary-network strategy's corresponding utility is traversed and calculated according to our utility definition by calling Alg. 1, thus the output is the maximum. ∎

Theorem 4.6: $(s_p^*, \boldsymbol{s}_s^*(s_p^*))$ is the equilibrium of the type-2 secondary-network follower case, where s_p^* and $\boldsymbol{s_s}^*(s_p^*)$ are defined in Lemma. 4.5 and 4.4 respectively.

Theorem 4.7: By playing type-2, the secondary network can guarantee it ends up with an equilibrium with higher or at least equal utility.

Proof: To prove this theorem, we only need to prove three facts: 1) under any primary network's strategy s_p, the secondary network could gain more utility by playing type-2 ($u_{s2}^*(s_p)$)than type-1 ($u_{s1}^*(s_p)$). This is because the type-2 secondary network doesn't need to consider its interference towards to primary network, while the type-1 secondary network needs to do so. Therefore the strategy space of the type-2's problem ORS2 is larger than that of type-1's problem ORS1; 2) with the increasing of primary network's stream number s_p, the secondary network's maximum utility $u_{s1/s2}^*(s_p)$ decreases. This is also due to the size of strategy space: enlarging primary network's sending-stream number will compress the secondary network's strategy space. 3) in equilibrium, the primary network always transmit larger or at least equal number of streams when coexisting with type-1 than type-2 network. This is obvious as the primary network always choose to send its maximum number of streams when coexisting with type-1 secondary network. Assuming s_{p1}^* and s_{p2}^* are the primary network's optimal strategies when dealing with type-1 and type-2 secondary networks respectively, then we have $s_{p1}^* \geqslant s_{p2}^*$, thus $u_{s1}^*(s_{p1}^*) \leqslant u_{s1}^*(s_{p2}^*)$. As we have $u_{s1}^*(s_{p2}^*) \leqslant u_{s2}^*(s_{p2}^*)$, thus we can have $u_{s1}^*(s_{p1}^*) \leqslant u_{s2}^*(s_{p2}^*)$. ∎



(a) (b)

Fig. 3: (a) single-link primary network with 2-flow multi-hop secondary network. (b) single-link primary network with 3-flow multi-hop secondary network

One straightforward insight from this theorem is that the secondary network has the incentive to violate the spectrum access rule. In the next section, we will use two case studies to verify this incentive and then explore the methodology to deal with such selfish secondary networks.

V. NUMERICAL RESULTS

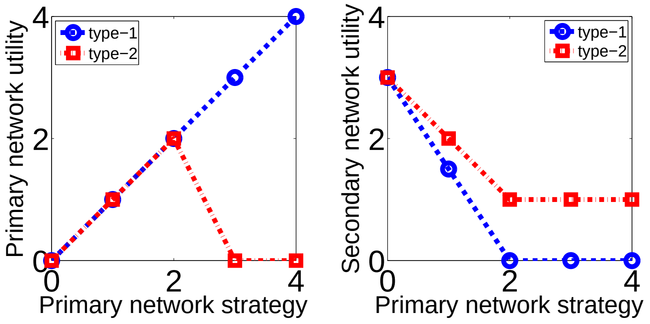
A. Overview

With our equilibrium calculating algorithms in Sec. IV, the primary network could estimate the equilibriums and the corresponding utilities. In this section, we will run our algorithms to show several results that provide insights of the coexisting game between primary and secondary networks. We choose 4 and 2 as the antenna numbers for primary and secondary networks respectively. We use two case studies in which the single-link primary network coexists with 2-flow and 3-flow multi-hop networks respectively. We will calculate and analyze both primary and secondary network's optimal strategies and the interference generated by secondary network's optimal response strategy. We will first show that the general multi-flow secondary networks always have incentives to play selfishly and non-compliantly, i.e., playing type-2, which is a bad news for the primary networks. Second, to deal with such selfish networks, we decompose our algorithms and analyze the intermediate status. Our analysis implies that by aggressively extending primary network's interfering range, we could enhance its own utility in the equilibrium.

B. Selfish Incentives of Secondary Networks

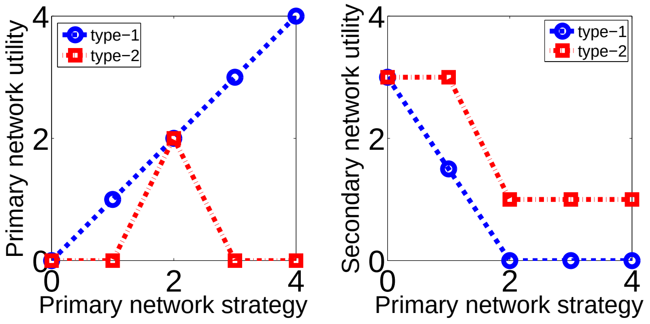
We will show that in general cases, the secondary networks always have the incentives to be selfish, i.e, playing type-2 rather than type-1.

To analyze the secondary network's selfish incentives, we compare its utilities by playing type-1 and type-2 respectively. We select a scenario (scenario 1 in Fig. 3 (a)) with



(a) (b)

Fig. 4: Primary (a) and secondary network (b) utilities under primary network's stream-transmitting strategies. In (a) the equilibrium of primary network is transmitting 4 streams with type-1 and 2 streams with type-2 secondary network.



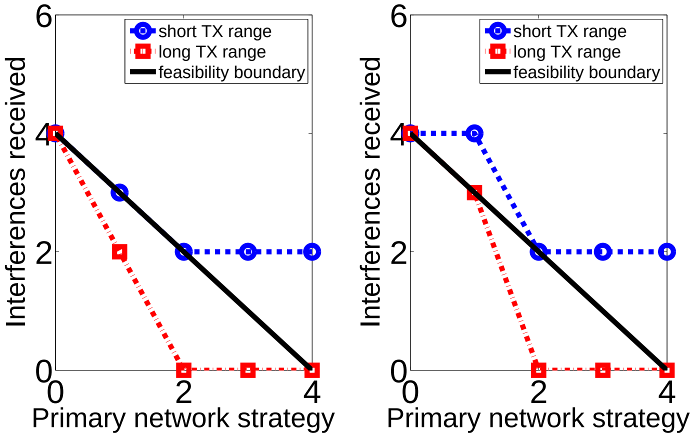
(a) (b)

Fig. 5: Primary (a) and secondary network (b) utilities under primary network's stream-transmitting strategies. In (a) the equilibrium of primary network is transmitting 4 streams with type-1 and 2 streams with type-2 secondary network.

a single-link primary-network flow and two single/multi-hop secondary-network flows. We run our algorithms and derive the equilibriums and the corresponding utilities. Specifically, when coexisting with type-1 secondary networks, the utilities of primary and secondary networks under equilibriums are $(4, 0)$. Meanwhile, the utilities are $(2, 1)$ when the primary network coexists with the type-2 secondary networks. To get more insights of the results, we insert breakpoints in line 2 and 4 of our Alg. 2 to observe the utilities under each of the primary network's stream-transmitting strategies. The results are shown in Fig. 4. From the results we can observe that 1) the primary network could always achieve higher or at least equal utilities when dealing with type-1 secondary network (as shown in Fig. 4 (a)) and the primary network's optimal strategies are transmitting 4 and 2 streams when coexisting with the two types of secondary networks respectively; 2) However, the secondary network could always obtain higher or at least equal utilities by playing type-2, i.e, being selfish and non-compliant. Similar results could be derived from another case-study (scenario 2) results shown in Fig. 5.

C. Countering Selfish Secondary Networks

As we have shown the secondary networks always have the incentives to behave selfishly, it is necessary for the primary network to protect its transmission against selfish secondary networks' interferences. However, the interference



(a) (b)

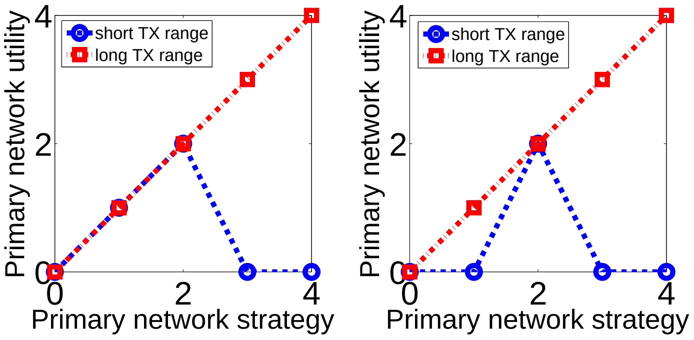
Fig. 6: Interference towards primary network under its stream-transmitting strategies with different primary network's TX-interference ranges. (a) scenario 1 (b) scenario 2.

from secondary networks is complex as it is implicitly affected by primary network's strategies. Using our MILP problems formulation, we could precisely quantify the interferences from secondary networks through intermediate status analysis of our Alg. 2. Our intuition is that the larger degree of interferences generated by primary networks (in our system model, the interference is controlled by the primary network's TX range and its transmitting strategies), the less interference it receives from the secondary networks in return. We validate our conjectures using our two case studies shown in Fig. 6.

In Fig. 6, we can observe that by transmitting more streams, the primary network endures less interference back from the secondary network. However in practice, the number of transmitted streams is capped by the antenna numbers of the primary network thus we cannot increase the transmitting-stream number arbitrarily. Another observation is that by using long transmitting range, the primary network could also reduce the interferences generated by the secondary network through a chain of interferences out to and back from it. Though we show only two cases with two different network settings, such a phenomenon can be observed in general cases. This is because longer transmission range means larger interference degree generated on the secondary networks, thus more DoFs are consumed to perform IC at the secondary network, which otherwise could be used to transmit more streams in the secondary network. Therefore less interferences are feed back to the primary network in return as the interference is the result of stream transmissions. The longer transmitting range could finally enhance the utilities of the primary network in the game equilibriums, which is validated in Fig. 7. Note that this transmitting-range-extending approach is different from the 'tit-for-tat' approach in repeated game, as we don't force the secondary network to change into the compliant type. Instead, by using this approach, we aim at improving the primary network's utility in the equilibrium when coexisting with the non-compliant secondary network.

VI. CONCLUSION

In this paper, we studied the coexistence problem in cognitive radio networks with MIMO capability using game-



(a)

(b)

Fig. 7: Primary network's utilities under its stream-transmitting strategies with different primary network'd TX-interference ranges. (a) scenario 1 (b) scenario 2. The equilibriums of primary network is transmitting 4 and 2 streams in long and short TX ranges in both scenarios.

theoretical approach. We formulate the game between the single-link primary network and the multi-hop secondary network as a Stackelberg game. To derive the equilibrium, we designed an algorithm based on the solution of a mixed-integer-linear-programming problem. Our results show that the multi-hop secondary network always has the incentive to play selfishly. The results also imply that we could enhance the primary network's utility by enlarging its transmitter's interference range. In future work, we will explore approximate algorithms to solve the optimization problem & game equilibrium more efficiently, and extend the framework to general multi-hop networks coexistence.

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